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GRAVITATION AND RELATIVITY

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Foreword

While astronomy and geology have traditionally been sciences involving observation and classification of phenomena in the universe, the other physical sciences have been largely restricted to laboratory investigations of the laws of nature and their manifestations in simple forms of matter. In recent years, however, immense progress has been made in understanding how the laws of nature operate in the universe itself—in the cosmic laboratory—where man cannot perform simple experiments but must attempt to analyze nature as he finds it. Progress has been particularly vigorous in such fields as astrophysics, geophysics, geochemistry, and meteoritics. In particular, the space research program has stimulated large numbers of people from various physical disciplines to participate in the physical exploration of the solar system.

This series of books will be concerned with any line of scientific inquiry which attempts to achieve a better understanding of the physical mechanisms that operate in the universe. Pure investigations of the laws of nature, and laboratory investigations of the properties of matter, will not be included. If a laboratory scientist turns his experimental and theoretical talents to the investigation of his physical environment, the results of his investigations are of interest for this series.

The primary aim of the series will be to further communication between scientists investigating nature, and the mode of publication will be varied to minimize the diversion of a scientist's energy from his active participation in teaching and research. The series will include monographs on various specialized topics, proceedings of conferences and symposia, collections of scientific reprints with critical commentary, and publication of lecture-note volumes.

A. G. W. Cameron
Series Editor

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The many faces of Mach

R. H. Dicke

Two Historical Viewpoints: Absolute Space and Relativistic Space

It is rather interesting that, as far as I know at least, only two pictures of physical space have ever been proposed. Even going back to the ancient Greeks, there has not appeared any other than these two pictures. One employs the idea of absolute space. This is the notion that a physical space has a structure of its own. From ancient times to the twentieth century, this was usually taken to mean that space was filled with a medium of some kind. This idea appeared in Descartes' philosophy. The medium, which he called a plenum, was presumed to carry the planets around the sun as ships floating in a gigantic vortex in the ethereal sea. This accounted for the motions of the planets, and in a completely quantitative way, provided one made the right assumptions about the vortex, namely, that it moved in just the way the planets are observed to move.

Newton's ideas concerning an absolute space are well known. His law of gravitation seems to imply an "action at a distance" in a vacuum. After a visit to England, Voltaire wrote in 1730, "A Frenchman who arrives in London will find philosophy like everything else very changed there. He left the world a *plenum* and now he finds a *vacuum*." However, Newton seems to have had an ether- (or plenum-) filled space in mind when he constructed his theory of gravitation. At one time[1] he said:

> That one body may act upon another at a distance through a vacuum without the mediation of anything else . . . is to me so great an absurdity that I believe that no man, who has in philosophical matters a competent faculty for thinking, can ever fall into it.

It is interesting that the other picture of space is also an old one, going back at least to the early eighteenth century. It seems to have first appeared in some statements made by the great British philosopher, Bishop Berkeley. This is the idea of a relativistic space. According to this idea, the only meaningful concepts of physical space are concepts that relate the position of matter relative to other matter. From this point of view empty space is devoid of sign posts, and the only properties space possesses are properties derived from the matter in the space.

I quote from Berkeley's writings as they appear in Sciama's book, *The Unity of the Universe.*[2] Similar statements can be found in *The Principles of Human Knowledge*, by Berkeley.[3]

> If every place is relative then every motion is relative, and as motion cannot be understood without the determination of its direction which in its turn cannot be understood except in relation to our or some other body. Up, down, right, left, all directions and places are based on relations and it is necessary to suppose another body distinct from the moving one.

That is a very clear statement of the principle of relativity. Berkeley further elaborates on it:

> Let us imagine two globes, and that besides them nothing else material exists, then the motion in a circle of these two globes round their common centre cannot be imagined. But suppose that the heaven of fixed stars was suddenly created and we shall be in a position to imagine the motion of the globes by their relative position to the different parts of the heaven.

Thus we have this rather early statement of the idea of relativity, which was later formulated independently by Mach and has become known as Mach's principle.

Mach's statement follows[4]:

> For me only relative motions exist. . . .When a body rotates relatively to the fixed stars, centrifugal forces are produced; when it rotates relatively to some different body not relative to the fixed stars, no centrifugal forces are produced. I have no objection to calling the first rotation as long as it be remembered that nothing is meant except relative rotation with respect to fixed stars.

This is the idea that rotation of a body relative to the fixed-star system is equivalent to a rotation of fixed stars about the body. They both represent the same relative motions.

Then we come to some remarks of Einstein on the role of Mach's principle in physics[5]:

> But in the second place the theory of relativity makes it appear probable that Mach was on the right road in his thought that inertia depends upon a mutual action of matter. For we will show in the following that, according to our equations, inert masses do act upon each other in the sense of the relativity of inertia, even if only very feebly. What is to be expected along the lines of Mach's thought?

Einstein has a list of three effects that he would expect to exist if Mach's principle were valid:

1. A body must experience an accelerating force when neighboring masses are accelerated, and, in fact, the force must be in the same direction as that acceleration.

In other words, suppose we imagine a body inside a hollow spherical mass shell. If one were to accelerate the shell suddenly, this acceleration of nearby matter would, on a miniscule scale, be somewhat like an acceleration of the whole universe relative to the body. As a consequence, an inertial force might be expected to act on the body and to produce an acceleration of the body in the same direction as the acceleration of the shell. This inertial force ought to appear under these conditions as a direct effect of Mach's principle. Einstein showed that this effect exists in the formal structure of general relativity.

2. A rotating hollow body must generate inside of itself a "Coriolis field," which deflects moving bodies in the sense of the rotation, and a radial centrifugal field as well.

We may give a qualitative argument for this point. Rotating a hollow spherical shell is in some sense the same as rotating the whole universe with all the matter it contains. Furthermore, a universe rotating relative to some "fixed" coordinate system, from the viewpoint of Mach, is completely equivalent physically to a rotation of the coordinate system relative to a fixed universe. We know that, in the latter case of the rotating coordinate system, Coriolis and centrifugal forces appear. So in the former case, in the same sense, a mass shell rotating relative to a fixed coordinate system should produce something like a Coriolis and a centrifugal field in this fixed system inside the shell. This effect was discussed by Thirring and Lense[6] within the framework of general relativity.

3. The inertia of a body must increase when ponderable masses are piled up in its neighborhood.

Einstein seems to have been mistaken about this effect appearing in his theory of general relativity. The equations that Einstein exhibits to illustrate this effect[5] have their particular form as the result of the selection of a particular coordinate system, and the effect is simply a coordinate effect, occurring for a particular coordinate system only. This choice is arbitrary and there are no observable effects to be observed in a laboratory from matter piled up in a spherically symmetrical way about the laboratory. Brans has examined Einstein's discussion of this effect.[7]

Thus, effects 1 and 2 are present in general relativity but 3 is not. We shall encounter more evidence for the profound influence of the ideas of Mach on Einstein's thinking about gravitation. Traces of Mach's principle appears all through general relativity.

On the other hand, I would contrast the previous antediluvian quotations from Bishop Berkeley concerning the meaning of relativity with a modern statement appearing in Synge's book, *Relativity, The General Theory*.[8] In the preface he writes:

> I am much indebted to the well known books of Pauli, Eddington, Tolman, Bergmann, Möller and Lichnerowicz, but the geometrical way of looking at space-time was constructed from Minkowski. He protested against the use of the word "relativity" to describe a theory based on an "absolute" (space-time), and, had he lived to see the general theory of relativity, I believe he would have repeated his protest in even stronger terms. However, we need not bother about the name, for the word "relativity" now means primarily Einstein's theory and only secondarily the obscure philosophy which may have suggested it originally. It is to support Minkowski's way of looking at relativity that I find myself pursuing the hard path of a missionary. When in a relativistic discussion I try to make things clearer by a space-time diagram, the other participants look at it with polite detachment and, after a pause of embarassment as if some childish indecency had been exhibited, resume the debate on their own terms.

Having thus rejected the relativity of Mach and Berkeley, he goes on to deny another familiar principle of gravitation theory.

> The Principle of Equivalence performed the essential office of midwife at the birth of general relativity, but, as Einstein remarked, the infant would never have got beyond its longclothes had it not been for Minkowski's concept. I suggest that the midwife be now buried with appropriate honours and the facts of an absolute space-time faced.

We have, in this very modern point of view of an outstanding expert in relativity, the return to the idea that we are dealing with an absolute space-time. From the viewpoint of Synge, general relativity describes the geometry of an absolute space. According to him, certain things are measurable about this space in an absolute way. There exist curvature invariants that characterize this space, and one can, in principle, measure these invariants. Bergmann has pointed out that the mapping of these invariants throughout space is, in a sense, labeling of the points of this space with invariant labels (independent of coordinate system). These are concepts of an absolute space, and we have here a return to the old notions of an

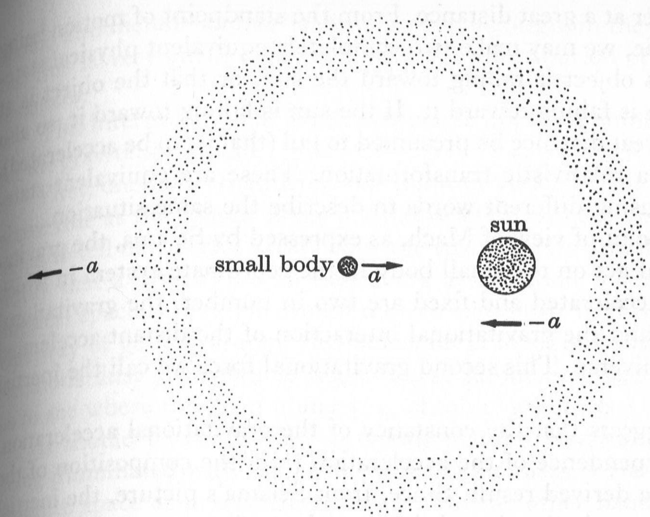

Figure 7-1 A simple model of the universe with a small body gravitating toward the sun. The rest of the matter in the universe is represented by a shell of matter at a great distance. It is possible to view the small body as accelerated relative to fixed matter at a distance and a nearly fixed sun or to view the matter at a distance and the sun accelerated relative to a fixed small body.

absolute space. According to this point of view, the chief difference between this absolute space and the one of Newton is that this space is four-dimensional and Riemannian, and we do not feel any necessity for filling it with a medium. Furthermore, the geometric properties of the space are influenced by the matter contained therein.

Sciama's Simplified Model for Interpreting Mach's Principle

The quotations we have presented indicate the extreme variety of the viewpoints of the experts in this field, including the giants of the past. It is this variety that motivated my choosing for a title of this lecture, *The Many Faces of Mach*. The face of Mach that I shall first present is that due to Sciama.[9] It is a greatly oversimplified picture, which discusses gravitation as a type of vector interaction. Similar discussions have been given by others. Recently, Weisskopf has based a similar discussion on the tensor interaction.

Imagine that we have a sun and some small test body falling toward it (Figure 7-1). We examine this in the framework of a universe with a large amount of matter at a great distance. From the standpoint of motion being purely relativistic, we may make two completely equivalent physical statements: that this object is falling toward the sun, or that the object is at rest and the sun is falling toward it. If the sun is falling toward it, so also may matter at great distance be presumed to fall (that is, to be accelerated). This is simply a relativistic transformation. These are equivalent statements. We are using different words to describe the same situation.

From the point of view of Mach, as expressed by Sciama, the gravitational forces that act on the small body in the coordinate system in which the body is unaccelerated and fixed are two in number, the gravitational pull of the sun and the gravitational interaction of the distant accelerated matter in the universe. This second gravitational force we call the inertial reaction.

Sciama suggests that the constancy of the gravitational acceleration, that is, the independence of the acceleration from the composition of the falling body, is a derived result. Since, from Sciama's picture, the inertial force is actually gravitational, and the special coordinate system is one for which the inertial (gravitational) force is balanced by the direct (gravitational) pull of the sun, this net force is zero and remains zero when some other test body is substituted. Thus, lead and aluminum weights would be expected to both remain at rest in this coordinate system and fall with the same acceleration in some other coordinate system.

According to Sciama, in this coordinate system the gravitational wave is radiated by the distant accelerated matter in such a way that the total force acting on the body is zero.

If one believes that this gravitational force can be interpreted as an ordinary force field that is propagated as a wave from great distance, there is a time retardation in this wave, similar to that which appears in the electromagnetic field of an accelerated charged particle. With such a wave phenomenon, the strength of the inertial reaction is expected to fall off as $1/r$, as one would expect from any time-retarded (radiation) effect.

According to Sciama's model, the inertial force on the small body shown in Figure 7-1, resulting from the acceleration of distant matter relative to it, is proportional to the mass of the small body m, the mass of the distant accelerating matter M, the value of the acceleration a, and the reciprocal of the distance to the accelerating matter:

$$F \approx \frac{GmMa}{rc^2} \tag{1}$$

The masses appear only in first order, since Sciama's formalism is linear. The r^{-1} dependence is resulting from the interpretation of this inertial field as a radiation resulting from the acceleration of distant matter. We should also include an inertial term having its origin in the accelerated sun, but this is very small compared with the contribution of matter at great distances.

This interpretation of Mach's principle implies a number of properties for the gravitational field.

First, one notes that, from this point of view, distant galaxies play an active role as the source of the inertial reaction and are not simply beacon lights to tell us where a "real" absolute space is. (One might say parenthetically that a difficulty with describing acceleration relative to an empty physical space is that one cannot discern sign posts in this absolute space. One way of getting around this objection is to simply distribute in space luminous dust of negligible mass. These luminous objects would permit us to see where the "real underlying absolute space" is.)

It should be noted that, in talking about a physical space as simply being illuminated with signposts of negligible mass, I am dealing with an absolute space as described by Synge. On the other hand, the picture of Berkeley, Mach, Einstein, and Sciama would suggest a much more direct physical relation between the inertial force and the presence of large amounts of matter at great distance. If one were to remove this matter, then according to Mach, the inertial force would disappear. If one were to reduce the matter to negligible proportions, there would be striking changes in local inertial effects. To summarize, according to Mach's point of view, we should interpret inertial effects as a consequence of interactions of matter at great distances in the universe with accelerated bodies in the laboratory. If we observe a gyroscope in the laboratory and this gyroscope continues to point to some particular distant galaxy, this is more than just an accidental correspondence. There are fields produced by matter at great distance, which interact with the gyroscope in the laboratory in such a way as to keep pointing in a direction fixed with respect to distant matter.

The model of Sciama is not a completely satisfactory framework for expressing Mach's principle. By selecting a particular coordinate system (with the test body at rest) for a discussion of gravitational effects, he has lost the advantage of general covariance. In attempting to express inertial forces as vector forces, the obvious difficulty encountered is that such forces are not acceleration-dependent. Thus, inertial effects must be described in Sciama's scheme of things in special coordinate systems where the test body is not accelerated. Also it is well known on other grounds that a vector theory of gravitation is unsatisfactory. For example, a vector interaction (such as an electromagnetic force) acts on a conserved quantity (such as charge), and the resulting accelerations depend upon the binding energy of a system of particles. Thus, in a vector theory, gravitational accelerations are not independent of the composition of the bodies.

Whereas Sciama's model of inertial effects does not provide a proper (coordinate-independent) theory of gravitation, it does provide a simple physical picture for the origin of inertial forces.

Limitations of General Relativity

As we saw from the statements of Synge, Einstein's theory is not relativistic in the Machian sense. In his theory, space has physical properties and constitutes a physical structure even in the absence of all matter. Motion of a massless test body is referred to as an absolute geometry.

On the other hand, it is remarkable how much of Mach's principle does permeate Einstein's general relativity. This will be discussed in detail later. Here we simply note that, by describing gravitational effects as resulting from a tensor field, Einstein was able to exhibit the inertial force, proportional to the acceleration of a particle, as one of the force terms derived from the tensor field. Nonetheless, general relativity does not appear to describe Mach's principle properly. This can be seen by noting that, in the absence of all matter, the metric tensor describes a flat space and that this flat space possesses inertial properties. Even Schwarzschild's famous solution is unsatisfactory, from the viewpoint of Mach. As one moves to infinity, and the mass source (the source of inertial forces according to Mach) disappears in the distance, the space becomes flat and continues to possess inertial properties in contradiction with the expectations of Mach.

It should be remarked, however, that a theory requires more than a set of differential equations. Boundary-value conditions, or initial-value conditions, are required before the theory is specified completely. Wheeler and others have remarked that the difficulties of general relativity *vis-à-vis* Mach's principle may be connected with boundary-value conditions, not field equations (see Chapter 15).

A Catalog of Long-Range Fields—Fermion Fields

As Mach's principle implies that distant matter exerts an influence on the laboratory, it is important to examine all the long-range fields with which we are familiar. These are the fields that could be the instruments through which such Machian interactions could take place.

Long-range interactions are caused by fields that, in the language of quantum-field theorists, represent zero-rest-mass particles. These fields can be cataloged as either boson or fermion fields asssociated with particles of integral or half-integral spin. To be of interest, these particles must be massless and chargeless.

Massless particles of half-integral spin such as neutrinos, do not provide a particularly promising way of getting gravitational effects. This

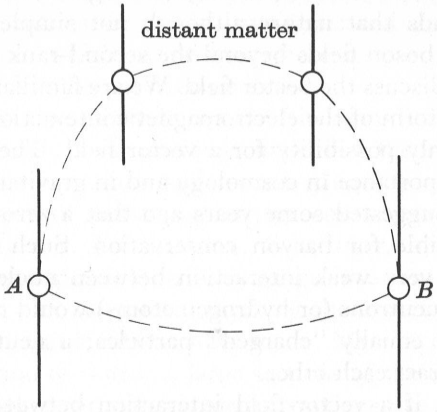

Figure 7-2 Feynman diagram of a four-neutrino exchange coupling bodies A and B and involving distant matter.

is because of the complexity resulting from the fact that at least two neutrinos are necessary in every static interaction. Neutrinos and antineutrinos must be paired properly.

Feynman, a few years ago, managed to obtain a $1/r^2$ force from an interchange of four neutrinos in the following way: A neutrino from body A is absorbed by a distant body; this body interchanges a neutrino with another distant body; a neutrino from this body is absorbed by body B; and, finally, a fourth neutrino is emitted from body B to body A completing the link (Figure 7-2). This leads to a $1/r^2$ force, between A and B, in a nice way.

Furthermore, there are some Machian ideas in this approach, because matter at great distance does play a role. There is a difficulty however. It turns out that nearby matter is more important than distant matter. Inertial effects are certainly not due mainly to the earth. We know that a gyroscope is observed to precess as the earth rotates. The interchange of four neutrinos does not seem to be a particularly promising approach to gravitation and to Mach's principle.

Long-Range Boson Fields—The Vector Field

The class of fields that appears to offer the most interesting possibilities for describing gravitational and inertial effects are long-range boson fields associated with neutral, zero-mass particles of integral spin. This class includes the scalar, vector, tensor, and higher-rank interactions. On the general grounds that nature although not simple is not vicious, I shall not consider boson fields beyond the second-rank tensor.

First, I shall discuss the vector field. We are familiar with a zero-mass vector field in the form of the electromagnetic interaction between matter. This is not the only possibility for a vector field. There could be other vector fields of importance in cosmology and in gravitation. For example, Lee and Yang[10] suggested some years ago that a zero-mass vector field might be responsible for baryon conservation. Such a field would be associated with a very weak interaction between nucleons. In the Lee-Yang theory, two neutrons (or hydrogen atoms) would repel each other as though they were equally "charged" particles; a neutron and an antineutron would attract each other.

One may ask if a vector-field interaction between particles of the same charge could be important on the scale of the universe. The answer seems to be that, if the hypothesis of a uniform and isotropic universe is valid and if the vector field is gauge-invariant, that is, if it obeys Maxwell-like equations, then, for large enough space averages, such a vector field cannot have important cosmological effects.[11] The reason for this is the following:

The antisymmetric tensor that represents electromagnetic fields (or the Lee–Yang vector field) has six nonzero components. These can be grouped into two sets of three components, both of which transform under proper rotations like ordinary spatial three-vectors and, in the electromagnetic case, are the electric and magnetic fields. If, in some coordinate frame, the universe appears isotropic in sufficiently large volume averages, there can be no net three-vector field averaged over such a volume. An "electric" or "magnetic" field would define a direction and destroy the isotropy. The electric and magnetic fields, when averaged over large volumes, must both be zero. Then all the components of the antisymmetric field tensor are zero, and, from the Maxwell-like equations these fields obey, the net charge and current densities must both average to zero over that volume.

$$F^{ij}{}_{;j} = J^i = 0 \tag{2}$$

This charge-current density is a four-vector. Hence, if its components are all zero in the special coordinate system that exhibits isotropy, they are zero in any coordinate system.

This is a quite general conclusion. If the assumption of isotropy and uniformity of the universe is a valid one, there can be no net "charge" or "current" over large-volume averages. If the Lee–Yang field were to exist, nucleons and antinucleons must be present in equal numbers over large volume averages. This seems unlikely. If it were the case, collisions between matter and antimatter would have generated a substantial amount of gamma radiation, which would contribute to the present cosmic-ray flux. Such a γ-ray flux has not been observed.

This argument does not rule out the Littleton–Bondi suggestion concerning a charge excess in the universe, since their theory involves a breakdown of Maxwell's equations and is not gauge-invariant. Nor does it deny the possibility of obtaining baryon conservation from a gauge-invariant vector theory. For instance, Schwinger, in a recent paper,[12] pointed out that it is possible to have gauge-invariant vector fields with nonzero rest mass. One cannot say anything about such short-range forces on the basis of the arguments we have made. My conclusion is that, with the usual assumption of isotropy, large sections of the universe cannot interact with each other through vector interactions obeying Maxwell-type (gauge-invariant) equations in a uniformly isotropic universe.

There is a more general result in the case of a closed universe if we deal with a vector interaction involving charge of only one sign. In this case, it is not necessary to assume isotropy or uniformity. In any closed universe, a gauge-invariant vector interaction that interacts with matter, all assumed to be of the same sign of charge, cannot exist. There can be no net charge in such a universe because there is no place for a net outward "electric" flux to go. All flux lines must terminate on an equal number of charges of the opposite sign. But the existence of equal numbers of opposite charges is contrary to our assumption that all matter is a source for this field of the same charge. Hence there cannot be a vector field that depends on all matter in a closed universe being a source of the same sign. From this and the previous argument, I conclude that the vector field does not provide a promising approach to an expression of Mach's principle.

The scalar and tensor fields do appear to be capable of providing long-range gravitational effects of importance to cosmology and Mach's principle. Next, I shall describe some of the properties of the scalar interaction; properties that are not as well known as might be expected.

The Scalar Field: Some Unfamiliar Properties

I shall introduce a scalar-field variable that is generated by the matter in the universe and that varies from point to point. A four-force on matter is given by the gradient of that scalar:

$$F_i = \phi_{,i} \tag{3}$$

It is rather remarkable that such an interaction of a scalar field with a particle treated relativistically cannot occur unless the mass of the particle is a function of the scalar. One can see this in the following physical way: One would expect the rate of change with respect to proper time of the three space components of the momentum of a particle interacting with a scalar field to satisfy the following equation:

$$\frac{\partial}{\partial \tau} P_\alpha = F_\alpha = \phi_{,\alpha} \tag{4}$$

The components $\phi_{,\alpha}$ are the spatial derivative of the scalar field and P_α are the corresponding momentum components.

Now consider the corresponding four-vector equation in the special case where the scalar field is a static scalar and the coordinate system is stationary so that the only nonzero components of the gradient are spacelike. Then the rate of change with respect to the time component is equal to zero.

$$\phi_{,4} = 0 \tag{5}$$

This means that the energy of the particle is equal to a constant [Eq. (4)].

$$E = m/\sqrt{1 - v^2/c^2} \tag{6}$$

On the other hand, Eq. (4) implies that, if the spatial derivatives are nonzero, the particle is accelerating. We have a situation in which the velocity of the particle is changing while the kinetic energy remains constant. Equations (5) and (6) can be satisfied only if the mass of the particle is changing. This appears to be a very general result. The interaction of a scalar field with a particle leads to a variation of the mass of the particle as a function of the scalar field.

Note, for example, that the obvious variational principle for obtaining equations of motion with a scalar interaction is

$$0 = \delta \int [\tfrac{1}{2} m u^i u_i + \phi]\, d\tau \tag{7}$$

where u^i is the four-velocity, m is the mass of the particle, assumed constant, and ϕ is the scalar-field variable. However, the Euler equation of motion obtained from this variational principle,

$$\frac{d}{d\tau}(m u_i) = \phi_{,i} \tag{8}$$

is not a correct equation. This can be seen by multiplying Eq. (8) by u^i and making use of the relation

$$u^i u_i = -1 \tag{9}$$

and the assumption that the mass is constant, to obtain the expression

$$u^i \frac{d}{d\tau}(m u_i) = \frac{1}{2}\frac{d}{d\tau}(m u^i u_i) = -\frac{1}{2}\frac{d}{d\tau}(m)$$
$$u^i \frac{d}{d\tau}(m u_i) = u^i \phi_{,i} = \frac{d\phi}{d\tau} \tag{10}$$

Therefore,

$$\frac{1}{2}\frac{d}{d\tau}(m) = -\frac{d\phi}{d\tau} \tag{11}$$

But Eq. (11) is inconsistent with the assumption that the mass is constant. A correct variational principle consistent with Eq. (9) is obtained by dropping the assumption that the mass is constant and making the variation subject to Eq. (9) as a constraint. Alternatively, it can also be obtained from the variational principle:

$$0 = \delta \int m\sqrt{-u^i u_i}\, d\tau \tag{12}$$

In either case the equation of motion is

$$\frac{d}{d\tau}(m u_i) + \frac{dm}{d\phi}\phi_{,i} = 0 \tag{13}$$

and the mass of the particle is a function of the scalar field.

The scalar field leads to some other interesting effects. Since it is a boson field, one can simply sum all the effects of matter at great distance. Let me imagine the following simplified equation, something like a wave equation, satisfied by a scalar field. The source of this field is some scalar measure of the mass density in the universe ρ, multiplied by an interaction constant A:

$$\Box\phi = \nabla^2\phi - \frac{1}{c^2}\frac{\partial}{\partial t^2}\phi = -4\pi A\rho \tag{14}$$

We use a model of the universe consisting of a single shell of mass M and radius R. With the boundary conditions that ϕ goes to zero as R approaches infinity, the solution to Eq. (14) is

$$\phi = A(M/R) \tag{15}$$

for the interior of the mass shell.

It should be recognized that these equations are greatly oversimplified. The effects of space curvature are not included, and the matter density ρ has not been carefully specified. As the field ϕ is a scalar, the matter density must be a scalar, and a suitable measure is provided by the contracted energy-momentum tensor of matter. It is interesting to note that the virial theorem can be used to show that the time-averaged integral over a stationary localized body of the contracted energy-momentum tensor is equal to the total energy of the body. Thus, as in the case of the gravitational field, the scalar field is determined essentially by the total mass of the body under those conditions when gravitation is important. If we consider the interaction of some nearby matter through the scalar interaction, we see that the scalar will be a function of two terms, the effect of some nearby bit of matter, plus the contribution of matter at great distance.

Another interesting property of a scalar interaction is that it leads to an attraction between two bits of matter. Also, the strength of this interaction is of the order of magnitude of the gravitational interaction. It is very difficult to get a strong interaction with a scalar field because the sum over all the matter in the universe is enormous compared with the effect of the nearby object. The distant-matter term tends to dilute the effect of nearby matter, so that nearby matter has a relatively weak effect. One can show that, independent of the form, and independent of any preconceived ideas we have about how strong the coupling to the scalar is, the interaction strength turns out to be that of the gravitational interaction, within perhaps a factor of ten.[13]

The scalar field leads to an attraction that has about the right strength for gravitation. Also it exhibits one of the features of Mach's principle—namely, the inertial mass of a particle at a particular point depends upon the distribution of matter about that point (Einstein's third property). But the scalar field alone cannot represent gravitation. Other things are wrong. The scalar-field theory generally leads to the wrong perihelion rotation of Mercury. Also, light is not deflected by the scalar field as it goes by the sun; we cannot construct a theory of gravitation from a scalar field alone.

The Tensor Field: A Bridge from Mach's Principle to Geometry

The tensor field exhibits more or less those properties that we would expect to be associated with Mach's principle. We should examine how many of these come about.

First, if we desire a theory in which only the relation of matter to other matter is important, we would expect to express this theory in generally covariant equations. Explicit coordinate systems that might represent some intrinsic properties of space itself should not be used.

A second point is that, if we consider the laws of physics in two coordinate systems, one accelerated locally relative to the other, then what is an inertial force in one coordinate system can be a gravitational force in the other. There is an interplay between inertial and gravitational forces. They are two manifestations of the same thing. Because of this, we may eliminate the following tempting generalization of classic laws:

In classic mechanics, we write a variational principle as

$$\delta \int [\tfrac{1}{2}mv^2 - \text{p.e.}]\, dt = 0 \tag{16}$$

We would be tempted, perhaps, to generalize this for the laws of motion of a particle in a gravitational field with two kinds of terms, a term that would lead to the inertial effects and a term that would lead to the gravitational field effects:

$$\delta \int [I + G]\, d\tau = 0 \tag{17}$$

This is just the approach that is wrong from the standpoint of Mach's principle. One ought to be able to shuffle inertial forces and gravitational forces back and forth with simple coordinate transformations. But, if the integrand in Eq. (15) is an invariant (assuming generally covariant equations) integrated along some invariant measure of time along the path, a transformation from one coordinate system to another will not mix the two terms I and G in the integrand. The division between inertial and gravitational effects will be permanent if we write the equations this way.

This suggests that, to get Mach's principle into the theory in such a way that what is inertial force for one coordinate system is a gravitational force for another, we must obtain both forces from a single invariant. If inertial forces are to be connected with accelerations, it is clear that this invariant must be quadratic in velocities. The simplest such invariant is one that involves a tensor field and is given by $g_{ij}u^iu^j$. It is not obvious at this point that g_{ij} has anything to do with the metric tensor. The g_{ij} may be any non-skew-symmetric tensor field. From this invariant, we can form a variation principle for which both inertial and gravitational forces come from the same term:

$$\delta \int g_{ij}u^iu^j\, d\tau = 0 \tag{18}$$

In this form, a coordinate transformation will transform one type of force into the other.

This suggests that gravitation should have something to do with tensors and tensor fields. Otherwise, we are in trouble with the interchange and interplay between inertial and gravitational effects.

If we have only a single tensor-field entry in our equations for the motion of all particles, it is a simple step to call this tensor the metric tensor and define the geometry of the space in such a way that this is the metric tensor of that geometry. This, from one point of view, is an arbitrary definition of what we mean by the geometry. It is a geometry in which the gravitational trajectories of structureless particles are defined to be geodesics. That is, these paths are given by the variational principle:

$$0 = -\delta \int g_{ij} u^i u^j \, d\tau \tag{19}$$

with

$$-d\tau^2 = g_{ij}\, dx^i\, dx^j = ds^2 \tag{20}$$

and

$$u^i = \frac{dx^i}{d\tau}$$

Thus

$$0 = \delta \int ds \tag{21}$$

In the case of a particle of zero rest mass, we have

$$ds = 0 \tag{22}$$

and

$$\int ds = 0 \tag{22a}$$

Such trajectories are called null geodesics of the geometry.

This definition of the geometry is possible only if the motion of (essentially point) particles in a gravitational field are uniquely given independent of their composition by equations of the form of Eq. (19). If one bit of matter (e.g., a neutron) moves on a certain path, and another (say, a hydrogen atom) starting from the same initial conditions departs from this path, it would be impossible to define both paths as geodesics. In this case it would not be possible to determine a unique geometry from the gravitational motions of particles.

From the point of view of Mach's principle, we do expect a unique space-time to be associated with gravitation. We have given a rough but not conclusive argument for this in terms of Sciama's model. The uniqueness of the gravitational acceleration, in other words, the space-time (geodesic) path, is something that one might expect as a result of the application of Mach's principle.

Thus, we are led in a more or less straightforward way from considerations of Mach's principle to the ideas that we have unique space-time paths for the motion of matter in a gravitating field, that we may represent this motion by equations involving a tensor field, that we get the equations of motion from a variational principle of the form of Eq. (18), and that we may define this tensor field to be the metric tensor of our geometry.

There remains the question whether meter sticks would really measure the distances given by this geometry, because we have defined the distances simply in terms of the space-time motion of particles. The following argument indicates that meter sticks do properly measure distances in this geometry.

Equation (18) represents a generalization of the corresponding equation from special relativity, the Minkowski metric tensor being replaced by the generalized symmetric tensor g_{ij}. If the classic equation of motion, obtained as an Euler equation from the variational principle, is to be consistent with a quantum-mechanical formulation of the same problem, the wave equation must be derived from a Lagrangian density, with the Minkowski tensor again replaced by g_{ij}. If all the field equations, including Maxwell's, are generalized in this way, Minkowski's metric tensor disappears completely, being replaced by g_{ij}.

It is always possible to choose a coordinate system such that g_{ij} is Minkowskian and has zero first derivatives at a point. As long as second and higher derivatives are sufficiently small and the measuring rod is sufficiently small, the equations of motions of all the particles and fields comprising the measuring rod are identical (for this coordinate system at this particular point) with those of special relativity. Thus it measures a space-like length interval correctly. Being an invariant measure of this interval, this measure holds for other coordinate systems also. We conclude, therefore, that under these conditions, meter sticks and clocks made of ordinary matter, that is, constructed of particles interacting with the strong interactions, and for which gravitational interaction self-energies are rather small, do, in fact, measure this geometry in a proper way.

It is concluded that the tensor field is essential for expressing the laws of gravitation in a manner compatible with Mach's principle. We have also seen that the long-range scalar field could have important cosmological effects and play an important role in relation to Mach's principle.

Manifestations of Mach's Principle by the Scalar Field

We consider now the problem posed by the Schwarzschild solution in general relativity. Any localized mass distribution, in a space otherwise empty, determines a gravitational field that describes a flat space asymptotically. No other reasonable boundary conditions have been found. But, such a flat space, with no nearby matter, has absolute inertial properties.

This paradox could be avoided if space were to close about any localized mass distribution. Such space closure might be achieved if a scalar field were present, in addition to the tensor field.

With masses of particles varying inversely as some positive power of a scalar field λ and the boundary condition $\lambda \to 0$ asymptotically for an open space (or $\lambda = 0$ somewhere inside a closed space), one has a mechanism for closing the space. A particle could not move into a region with $\lambda \to 0$, for its energy would go to infinity. This suggests therefore that a scalar field with

$$m = m_0 \lambda^{-n} \tag{23}$$

is a possible way of achieving space closure.

Because of the great accuracy of the Eötvös experiment, it is necessary to assume the same λ dependence for all particles. Otherwise, a constant gravitational acceleration independent of composition would not be achieved.

Another feature of the scalar field is also useful for more fully incorporating Mach's principle into a relativistic theory of gravitation. As we have seen, the ideas of Berkeley, Mach, and Sciama lead us to believe that the gravitational acceleration of a body should depend upon the mass distribution about the body. We have noted (page 126) that Sciama's expression for the inertial force is

$$F \sim \frac{GMma}{Rc^2} \tag{24}$$

where, in this case, we take M to be the total mass of the universe to its visible limits and R to be a suitable measure of the radius of the universe. This equation agrees with the more usual expression

$$F = ma \tag{25}$$

only if

$$\frac{GM}{Rc^2} \sim 1 \tag{26}$$

This relation is also obtained from a consideration of the Lense-Thirring effect of general relativity. Inside a hollow massive shell, of mass m and radius r, rotating about an axis with angular velocity ω_1, a gyroscope would be expected to precess relative to distant matter in the universe at a rate

$$\omega_2 = \left(\frac{8}{3}\right)\frac{Gm}{rc^2}\omega_1 \tag{27}$$

If Mach's principle is applicable, ω_2/ω_1 should depend upon the mass distribution in the universe:

$$\frac{\omega_2}{\omega_1} = f\left(\frac{m}{M}, \frac{R}{r}\right) \tag{28}$$

where M and R are again suitable measures of the mass and radius of the observed portion of the universe. For $(m/M) \ll 1$ and $(m/r)(R/M) \ll 1$, the above equation, (27), due to Thirring and Lense, suggests that the appropriate lowest order expansion of the function f is

$$\frac{\omega_2}{\omega_1} = \gamma \frac{m}{M}\frac{R}{r} \tag{29}$$

where γ is a dimensionless number of the order of unity. Note that, when combined with Eq. (27), this gives

$$\frac{GM}{Rc^2} \sim \gamma \tag{30}$$

From a slightly different orientation and from the viewpoint of Mach the Coriolis forces appearing in a rotating coordinate system may be considered due to a Lense–Thirring rotation induced by the whole universe rotating about the point in question. If one were to naively apply the Lense–Thirring formula to this situation (for which it is not valid because it was derived from the weak-field solution) one would obtain

$$\omega_1 \sim \frac{GM}{Rc^2}\omega_1 \tag{31}$$

again implying that

$$\frac{GM}{Rc^2} \sim 1 \tag{32}$$

If an equation such as (32) is to be considered the prototype of a proper equation to be formulated exactly someday, either: (1) The mass distribution M/R is fixed by the field equation and/or by boundary conditions on the field equations or, (2) the gravitational constant is variable, being a function of the mass distribution.

Consider the first alternative. If Einstein's field equation is valid, the gravitational constant is fixed. If this be assumed, Eq. (32) can be incorporated into gravitation theory by introducing a scalar field. A scalar field, generated by all the matter in the universe and acting on the particles in the universe, could conceivably affect all their masses in such a way as to keep M/R constant. It should be noted that this would not represent a limitation upon the number of particles in the universe, but rather that, through the intercession of the scalar field, the masses of the particles would adjust themselves appropriately, in such a way as to give M/R the appropriate value.

It is as though the universe is a giant servosystem, continuously and automatically adjusting particle masses to the value appropriate to the feed-back condition

$$\frac{GM}{Rc^2} = 1 \tag{33}$$

In this connection, it should be remarked also that this effect of a long-range scalar field provides a ready explanation for the great weakness of gravitation, as expressed by the small size of the gravitational coupling constant compared with atomic coupling.

$$\frac{Gm_p^2}{\hbar c} \sim 10^{-40} \tag{34}$$

According to this explanation, this number is so small because m_p, the mass of the proton, has been reduced to very small value through the effect of the scalar field generated by the enormous amount of matter in distant parts of the nucleus.

In this connection, it should be remembered that, expressed in proton mass units, the mass of the universe is roughly

$$\frac{M}{m_p} \sim 10^{80} \tag{35}$$

and that the radius of the universe in Compton wavelengths is roughly

$$\frac{Rm_pc}{\hbar} \sim 10^{40} \tag{36}$$

The fact that

$$\frac{Gm_p^2}{\hbar c} \sim \left(\frac{m_p}{M}\right)^{1/2} \sim \frac{\hbar}{Rm_pc} \tag{37}$$

suggested to Dirac[14] that, as the age of the universe $\sim R/c$ changes, M and G change in such a way as to preserve this relation. In terms of the effect of an ordinary scalar field, we would say that m_p would change rather than G.

The author has argued[15] that the two relations given by Eq. (37) may be obtained from Eq. (32), which is implied by Mach's principle and a consideration of the conditions necessary for the existence of the solar system.

It is suggested that general relativity may be made compatible with the requirements of Mach's principle if a long-range scalar field exists in addition to the tensor field of Einstein's theory.

Such a scalar field could serve to close the space about a localized mass distribution and to so adjust particle masses as to satisfy the Machian condition

$$\frac{GM}{Rc^2} \sim 1 \tag{32}$$

Also, in accordance with Einstein's 3rd requirement discussed above, an effect not present in ordinary general relativity, a scalar field of the type to be described and generated by matter piled up about a laboratory serves to increase inertial effects. The chief observable effect of these increased inertial forces is a reduction in locally induced gravitational accelerations. (That is, because of increased inertial reactions gravitational accelerations are less.)

In Chapter 8, I shall indicate that the appropriate scalar theory is one for which particle masses vary as $\lambda^{-1/2}$, and that the theory may be transformed, with a unit transformation, in such a way that particle masses are constant but that the gravitational "constant" is variable. In this form of the theory, the gravitational coupling constant $Gm_p^2/\hbar c$ varies as a result of G varying. m_p, $\hbar$, and c are all constant. With this form of the theory, the Einstein field equation is not valid. Instead, one has an equation that is essentially one of the Jordan equations. This point will be discussed more in detail in Chapter 8. In Chapters 8 and 12, I shall consider the implications for astrophysics and geophysics of the scalar field, should it be found eventually to exist.

In summary, we have seen that there are many faces of Mach, almost as many as there are people who have written on the subject. Grounded as it is in philosophical matters of deep significance, the principle is *anschaulich* and difficult to raise (or lower, depending upon your point of view) to the status of a quantitative theory. It is suggestive that Einstein was lead to his very elegant theory of gravitation by hints derived from this principle. It may still contain much of value to future physicists.

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