Philosophy 421 — R. Rynasiewicz
Mathematical Logic II — Spring 2022

Homework # 1

Due Tuesday, February 22 (100 points, 10 points each)

1. Show that the set of wff's of a countable language is countable.
2. Show that the theory of dense linear orderings without endpoints is $\aleph_0$-categorical, i.e., that any two countable models are isomorphic.
3. In our discussion of non-standard models of arithmetic, we invoked compactness in claiming that, because every finite subset of $\Theta \cup \mathrm{Th}\ \mathfrak{N}$ has a model, $\Theta \cup \mathrm{Th}\ \mathfrak{N}$ also has a model $\mathfrak{A}$. But we also claimed that $|\mathfrak{A}|$ can be taken to be countable. Justify this latter claim.
4. Show that Robinson's **Q** is a subtheory of **PA**.
5. Show that for all $m, n \in \mathbb{N}$, CA $\vdash \overline{m} \boldsymbol{+} \overline{n} = \overline{m+n}$. *Suggestion*: First take m to be an arbitrary natural number. Then do the induction on the parameter n.
6. Show that for all $m, n \in \mathbb{N}$, CA $\vdash \overline{m} \bullet \overline{n} = \overline{m \cdot n}$.
7. Show that for any variable free term τ, CA $\vdash \tau = \overline{n}$, where n is the denotation of τ in $\mathfrak{N}$ (i.e., where $\overline{s}(\tau) = n$ for any mapping s of variables into $|\mathfrak{N}|$). *Suggestion*: Do induction on the shape of τ.
8. Show that for any quantifier free sentence σ in the language of arithmetic both

   $$\mathrm{CA} \vdash \sigma \Leftrightarrow \sigma \in \mathrm{Th}\ \mathfrak{N}$$

   and

   $$\mathrm{CA} \vdash \neg\sigma \Leftrightarrow \sigma \notin \mathrm{Th}\ \mathfrak{N}.$$

9. Show that for any quantifier-free sentence σ, either PA $\vdash \sigma$ or PA $\vdash \neg\sigma$.
10. Show that the set of quantifier free arithmetic truths is decidable.