Philosophy 421 — R. Rynasiewicz
Mathematical Logic II — Spring 2022

Homework # 3

Due Tuesday, March 29 (100 points)

1. Show that, in any non-standard model of Presburger arithmetic, between any two Z-chains there exists a third. [10 points]

2. Show that Robinson's Q is "order adequate", i.e., establish the following nine propositions. *Hint*: For all except the first, proceed using either weak or strong numerical induction on the parameter n. [10 points each]

   (a) $Q \vdash \forall x\,(0 \leq x)$

   (b) For each n, $Q \vdash \forall x\,(x = 0 \vee x = 1 \vee \ldots \vee x = \bar{n} \rightarrow x \leq \bar{n})$

   (c) For each n, $Q \vdash \forall x\,(x \leq \bar{n} \rightarrow x = 0 \vee x = 1 \vee \ldots \vee x = \bar{n})$

   (d) For each n, if $Q \vdash \varphi(0)$, $Q \vdash \varphi(1)$, ..., and $Q \vdash \varphi(\bar{n})$, then $Q \vdash (\forall x \leq \bar{n})\varphi(x)$

   (e) For each n, if $Q \vdash \varphi(0)$, or $Q \vdash \varphi(1)$, or ..., or $Q \vdash \varphi(\bar{n})$, then $Q \vdash (\exists x \leq \bar{n})\varphi(x)$.

   (f) For each n, $Q \vdash \forall x(\bar{n} \leq x \rightarrow \bar{n} \leq Sx)$

   (g) For each n, $Q \vdash \forall x(\bar{n} \leq x \rightarrow \bar{n} = x \vee S\bar{n} \leq x)$

   (h) For each n, $Q \vdash \forall x(x \leq \bar{n} \vee \bar{n} \leq x)$

   (i) For each $n > 0$, if $Q \vdash (\forall x \leq \overline{n-1})\varphi(x)$, then $Q \vdash (\forall x \leq \bar{n})(x \neq \bar{n} \rightarrow \varphi(x))$