Philosophy 421 | R. Rynasiewicz
Mathematical Logic II | Spring 2022

Homework # 4

Due Tuesday, April 19. 70 points total (10 each).

1. Let $P(x)$ be a primitive recursive property. Show that $(\exists x \leq \overline{n})P(x)$ is also primitive recursive.
2. Let $f(n) = 5 + n$.
   (a) Give a primitive recursive definition of f.
   (b) Let $k_m = f(m)$. Consider the sequence k_0, k_1, k_2. Find numbers c and d such that $\beta(c, d, i) = k_i$ for $i = 0, 1, 2$.
3. Let $f : \mathbb{N} \to \mathbb{N}$. Show that if the theory T is at least as strong as Calculator Arithmetic, then wff $\varphi(v_0, v_1)$ represents f in T iff $\varphi(v_0, v_1)$ functionally represents f in T.
4. Show that 0, S, and the projection functions are representable in Q.
5. Find a predicate $ModusPonens(v_0, v_1, v_2)$ that defines the relation $\{\langle \#\varphi, \#\psi, \#\chi \rangle \mid \langle \varphi, \psi, \chi \rangle \in MP\}$ and show that the relation is p.r.
6. Find a predicate $ProofSeq(v_0)$ that defines the property $\{\mathcal{G}(d) \mid d \text{ is a proof sequence in PA}\}$ and show that the property is p.r.
7. Find a predicate $Proof(v_0, v_1)$ that defines the property $\{\langle \mathcal{G}(d), \#\varphi \rangle \mid d \text{ is a proof in PA of } \varphi\}$ and show that the relation is p.r.