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CHANGE AND MOTION

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Throughout the Middle Ages, motion and change were seen as the fundamental and immediate expressions of the innate natures of physical things. To understand their causes and effects was to grasp the nature of physical reality and to approach an understanding of higher realities such as spiritual substances and God. Philosophical treatments of motion and change in the Middle Ages arose out of teaching and commenting on Aristotle's natural works (the *libri naturales*) by masters in the medieval universities. Although banned several times at Paris early in the thirteenth century, by mid-century these works came to constitute almost the entire program in natural philosophy (*physica* or *philosophia naturalis*), which, along with logic, moral philosophy, and metaphysics, made up the core of the medieval arts curriculum. Beyond the faculty of arts, theories of motion and change came to be applied to theological problems such as grace and the sacraments, and they were in turn influenced by theological and other considerations. Motion and change were matters of intense interest to medieval natural philosophers, logicians, mathematicians, and theologians alike, and the scope of their studies extended far beyond the narrow limits of what in the seventeenth century would become dynamics and kinematics. Within medieval treatments of motion and change, there emerged a number of scientific and mathematical advances of great interest and lasting importance in the history of science, including a rule on the relation between force and speed, impetus theory, the quantification of qualities, the mean-speed theorem, and the graphical representation of qualities and motion. The medieval science of motion, therefore, was not merely an imperfect foreshadowing of the physical science of Galileo and Newton but a comprehensive program of natural philosophy, the purpose of which was a complete understanding of the natural world – the world of motion and change – through its principles and causes.

Aristotle's *libri naturales* covered the whole of physical reality and all the particular kinds of natural bodies, with their various motions and changes, including the heavens, elemental bodies, atmospheric and terrestrial phenomena, and living things. Of these, by far the most important within the medieval arts curriculum was his *Physica* (*Physics*), in which Aristotle gave an account of the general principles of nature, motion, and change. Here he discussed the nature of motion and change and their kinds, as well as time, place, void, continuity, speed, the relation between movers and moved bodies, self-movers, natural and forced motions, projectiles, and the first motion and first mover. Of the books on particular branches of natural philosophy, *De caelo* (*On the Heavens*) treated the nature and motions of the celestial realm and contained Aristotle's most extensive treatment of natural and forced motions, the motions of heavy and light bodies, and the nature and motions of celestial bodies. Together, *Physics* and *De caelo* thus provided the framework and most of the material for medieval discussions of motion and change.[1]

CHANGE AND MOTION

Change and motion are as puzzling to the mind as they are obvious to the senses. Most philosophers before Aristotle recognized their existence in the sensible world but were unable to explain satisfactorily what they were or their relation to other existing realities. The pre-Socratic philosopher Parmenides argued against the reality of all change on the grounds that nothing can come from nothing and that what already exists cannot come to be. Aristotle, for his part, dismissed such denials of the obvious and sought instead to account for motion and change through the discovery of the general principles of natural things. A principle of something is its source or origin, that from which it comes or what makes or constitutes it. In the first book of his *Physics*, Aristotle concluded that three principles underlie the existence and motion of natural things: the most general pair of contraries, called form and privation, and the subject that underlies them, called matter. Form is whatever qualifies a thing, makes it what it actually is – for example, white or hot. Privation, by contrast, is the exact absence of that form, what the thing could be but is not – black or cold. And matter is the underlying subject that receives or is subjected to different forms. The idea of form and privation thus breaks the Parmenidean deadlock by allowing a thing both to be in some sense (to be white or hot) and not to be in another (not to be black or cold). Motion, then, is the transition of a subject from privation to form or

[1] On motion and change in general in the Middle Ages, see John E. Murdoch and Edith D. Sylla, "The Science of Motion," in *Science in the Middle Ages*, ed. David C. Lindberg (Chicago: University of Chicago Press, 1978), pp. 206–64; and James A. Weisheipl, *The Development of Physical Theory in the Middle Ages* (1959; repr. Ann Arbor: University of Michigan Press, 1971). Many of the original texts are edited and translated, with commentary and introductions, in Marshall Clagett, *The Science of Mechanics in the Middle Ages* (Madison: University of Wisconsin Press, 1961), pp. 163–625, and Edward Grant, *A Source Book in Medieval Science* (Cambridge, Mass.: Harvard University Press, 1974), pp. 228–367.

from form to privation that occurs successively – over time – rather than all at once.[2]

Aristotle took it as obvious that all motion and change proceeds from something (some condition or place) toward something. The goal to which the motion and change of a natural thing is directed is determined precisely by its *nature*. The nature of a seed is to grow into a particular sort of plant; the nature of a heavy body is to descend. The nature of a thing is thus its internal source or principle of change or motion toward its natural goal. This nature is often associated with its matter (as the nature of a tree is sometimes said to be wood), though it is more properly to be associated with the form, which makes the thing what it is and gives it its proper propensity to move.[3]

In addition to matter and form, which medieval commentators often called the internal causes of a thing, Aristotle had distinguished two external causes: the efficient cause and the final cause. The efficient cause is what effects the change or impresses a form on matter; in Aristotle's example, the sculptor is the efficient cause of the statue. The final cause is the purpose or end – that for the sake of which. Thus the final cause of the statue is to be venerated in a temple. Natural things, as well as artificial things such as statues, can be accounted for by all four causes, though in natural things the formal, efficient, and final causes often coincide. The final cause also often determines the others in that ends often determine means, and for this reason Aristotle considered the final cause to be the most important in nature. As a result, his physics is often called *teleological* (from the Greek *telos*, meaning end or goal). According to Aristotle, nature – the internal source or principle of motion in natural things – acts for a purpose.[4]

Aristotle distinguished the motion and change that happens by nature from what is forced on a body by an external agent in opposition to its natural tendencies. Such motion was called violent or forced or compulsory, obvious examples of which are heavy bodies forced upward and projectiles propelled by a sling or bow. Unlike natural motions, which tend to be slower at the beginning and faster at the end, violent motions are usually faster at the beginning, when the external cause is strongest, and slower at the end, when the natural tendency of the body has taken over. In addition to natural and violent motions, Aristotle discussed motions and changes that are the result of willed or deliberate action, which medieval commentators called "voluntary" motions, thus distinguishing, according to cause, three kinds of motion: natural, violent or compulsory, and voluntary.[5] The medieval natural philosopher sought to distinguish these kinds of motions from one

[2] Aristotle, *Physics*, I.7.189b30–190b17; and Aristotle, *Metaphysics*, XI.9.1065b5–1066a34.
[3] Aristotle, *Physics*, II.1.192b8–193b21.
[4] Ibid., II.3–9.194b16–200b8.
[5] James A. Weisheipl, "Natural and Compulsory Movement," *New Scholasticism*, 29 (1955), 50–81, reprinted in James A. Weisheipl, *Nature and Motion in the Middle Ages*, ed. William E. Carroll (Washington, D.C.: Catholic University of America Press, 1985), pp. 25–48.

another and to determine their proper and immediate causes. This will become especially apparent later in this chapter in the discussions of motion in a void, projectiles, falling bodies, and celestial motions.

Whatever their causes, motion and change are of several different kinds, so one must ask whether these are fundamentally different or fundamentally the same, and whether motion itself is some reality apart from the subject in motion, on the one hand, or the goal or terminus of the motion on the other. For Aristotle, motion or change – both words translate his *kinesis* and henceforth will be used interchangeably – was not a reality itself but an actuality occurring in relation to quantity, quality, and place.[6] Thus he distinguished three kinds or species of motion: change of size or quantity (augmentation and diminution in the case of living things, rarefaction and condensation in the case of nonliving things), change of quality (or alteration), and change of place (local motion). A fourth candidate – substantial change (the transformation of one kind of thing into a completely different kind of thing) – Aristotle eliminated as a kind of motion since it occurs suddenly rather than over time. His medieval commentators called substantial change mutation (*mutatio*), and some at least included it in their discussions of motion in general.

To confuse the matter, Aristotle had said in his *Categories* that being heated and being cooled were examples of *passio* (being acted upon), which implied that all motion is a single kind of thing – a *passio* – distinct from the body in motion and its terminus.[7] The eleventh-century Arabic commentator Avicenna (Ibn Sīnā, 980–1037) took up this implication, holding that motion in general was a *passio* with four species or kinds (local motion and qualitative, quantitative, and substantial change), and distinct both from the body in motion and its terminus. Averroes (Ibn Rushd, 1126–1198), who was known in the Latin West as "The Commentator" for his exhaustive commentaries on Aristotle, tried to resolve the apparent contradiction in Aristotle by conceding that although motion was a *passio* formally, it was of four kinds materially, and in this respect motion belongs to the same species as its terminus.[8]

The question of the nature of motion was among the many physical problems first treated in a thorough, scholastic manner in the Latin West by Albert the Great (Albertus Magnus, ca. 1200–1280), in his commentary on *Physics*, and his discussion largely set the terms for later debate. Albert was especially concerned with distinguishing the truth of Aristotle's views from what he saw as distortions and misinterpretations introduced by Avicenna and Averroes, whose commentaries had accompanied the works of Aristotle in their reception in the Latin West. Aristotle had defined motion as

[6] Aristiotle, *Physics*, V.1–2.224a21–226b17; and Aristotle, *Metaphysics*, XI.11–12. 1067b1–1069a14.

[7] Aristotle, *Categories*, 9.11b1–5.

[8] Ernest J. McCullough, "St. Albert on Motion as *Forma fluens* and *Fluxus formae*," in *Albertus Magnus and the Sciences*, ed. James A. Weisheipl (Toronto: Pontifical Institute of Mediaeval Studies, 1980), pp. 129–53 at pp. 129–35.

"the actualization of what exists potentially insofar as it is potential," a definition that gave medieval commentators far less trouble than it later gave René Descartes (1596–1650), who famously declared it to be incomprehensible. It was obvious to Aristotle that motion was some kind of actuality since a body in motion is actually different from one at rest. But the crucial characteristic of motion is that, while it is occurring, it is incomplete. Motion is an actuality, but an incomplete actuality (what we might call a process), for before the motion has begun, the body is merely potentially in a new place, and once it has arrived in that new place its actuality is complete. Only while that potentiality is actually being realized precisely as potentiality is there motion. Motion is thus an incomplete actuality, the actualization of what exists potentially insofar as it is potential. Medieval commentators called this definition the formal definition of motion to distinguish it from what they called its material definition: that motion is the actuality of the mobile body precisely as mobile.[9]

For Albert, as for his student Thomas Aquinas (ca. 1224–1274), motion was not a univocal term – having exactly the same meaning in its various applications and referring to a single kind of being – but an analogous term, in that it is applied by analogy to more than one kind of being. Thus each different motion is in fact the same in essence as its terminus or end, differing from it only in perfection; for example, the motion of becoming white is in its essence the same as whiteness – its goal and end – except less perfect or complete (i.e., less white). According to Albert, then, motion is the continuous going forth of a form (*continuus exitus formae*) from mover to mobile, a form (e.g., whiteness) that flows rather than a static one. Thus Albert gave what he called the "total definition" of motion as the simultaneous activity of both mover and moved body, and in this way he encompassed the external causes – the efficient and, at least implicitly, the final cause. Although the phrase was not used by Albert, motion under the view that it was identical in essence or definition to its terminus came to be called a *forma fluens* (a form flowing), the view almost universally accepted among later commentators, in contrast to a *fluxus formae* (the flow of a form), which later came to characterize Avicenna's position. But the phrases *forma fluens* and *fluxus formae*, however convenient for modern historians, were not used by medieval commentators with sufficient consistency and precision to delineate accurately their various positions concerning the nature of motion.[10]

The view of motion as a *forma fluens* was pushed to its extreme form by the relentless logical analysis of terms and the spare, nominalist ontology of the Oxford logician and theologian William of Ockham (1290–1349). For

9 Aristotle, *Physics*, III.1–3.200b12–202b29.
10 McCullough, "St. Albert on Motion as *Forma fluens* and *Fluxus formae*," pp. 135–53.

Ockham, there are no realities beyond absolute things (*res absolutae*) – individual substances and the qualities that inhere in them. Quantity, relation, place, time, action, passion, and the other Aristotelian categories are not absolute things but merely convenient ways of speaking that can be reduced to statements about absolute things. Likewise motion is not some kind of reality beyond the mobile body and the accidents or places that the body acquires successively without pausing in any. Although later natural philosophers did not always accept this conclusion, in general they formulated the question of the nature of motion in the same way as Ockham had.[11]

PLACE AND TIME

The genius of Aristotle's definition of motion is that it appeals only to prior and more general metaphysical principles – actuality and potentiality – and does not depend on preexisting or subsisting space and time, in which or through which bodies are supposed to move. Rather, three-dimensional extension is the first, essential, and fundamental property of all bodies. The essence of a body is to be extended, and to be extended is to be a body. Space, according to Aristotle and many of his medieval commentators, was a mental abstraction of the extension of bodies, a result of considering the three-dimensionality of a material body abstracted from its matter and all other forms. But the three-dimensionality of a body no more exists separately from that body than does its weight, though both can be thought of as though they did. The science that treats weight as separate from material bodies is statics or, as it was known in the Middle Ages, the science of weights (*scientia de ponderibus*); the science that treats extension in general as separate from material body is geometry, whereas stereometry treats specifically three-dimensional extension. To maintain that space is a subsisting, three-dimensional reality somehow separate from bodies not only results in certain physical absurdities (for example, that each part of space is inside another and so on to infinity) but also fails to explain how a body can move from one place to another. In Book IV of *Physics*, after rejecting this naive but common idea of empty space, Aristotle defined place (*ho topos*, Latin *locus*) in such a way as to avoid these absurdities while allowing for local motion. The place of something, he concluded, is the innermost, motionless surface of the body immediately surrounding it. The immediate and proper place of Socrates, for example, is

[11] James A. Weisheipl, "The Interpretation of Aristotle's *Physics* and the Science of Motion," in *The Cambridge History of Later Medieval Philosophy*, ed. Norman Kretzmann, Anthony Kenny, and Jan Pinborg (Cambridge: Cambridge University Press, 1982), pp. 521–36; and Anneliese Maier, "The Nature of Motion," in *On the Threshold of Exact Science: Selected Writings of Anneliese Maier on Late Medieval Natural Philosophy*, ed. and trans. Steven D. Sargent (Philadelphia: University of Pennsylvania Press, 1982), pp. 21–39.

the surface of the air touching him and of the ground touching the soles of his feet. Place is thus not some empty, separate three-dimensionality but the surface of a real body, which can be left behind and subsequently be occupied by some other body (for example, the air that flows in when Socrates walks away). Nor is place a mere mathematical abstraction; it is the real physical relation between bodies expressed as their surfaces in contact. This is crucial for Aristotle's physical theory because it explains the natural distinction between up and down that he attributes to place as well as the curious power that place has in the natural motions of elemental bodies.[12]

According to Aristotle, each of the four primary bodies or elements (earth, water, air, and fire) has its own natural place, the place to which it is naturally moved and in which it is naturally at rest. The place of earth is at the center of the cosmos, and that of fire is immediately below the sphere of the Moon. Between them are the concentric places of water and air. Fire has the natural tendency, called lightness or levity, to rise to its natural place at the circumference of the cosmos, whereas earth has the natural tendency, called heaviness or gravity, to sink to the center. The two other elements have a tendency either to rise or to fall depending on where they find themselves, as well as a certain susceptibility in general to be moved. Mixed bodies – those composed of several elements – have the tendency of the element predominant in them. The place of an elemental body is determined by the body that surrounds it and determines in turn whether it will be at rest (if it is in its natural place) or in motion (if it is not) and in which direction. Place, then, has within itself the distinction of up and down and the power or potency to actualize a body's natural propensity for rest or for motion up or down. Without place, there could be no distinction between natural and forced motions. Aristotle's definition of place was defended by almost all medieval commentators against what was taken to be the unlearned view that place was a three-dimensional reality somehow distinct from the bodies in it – what we now call "space."[13] The possibility of the separate existence of such a three-dimensional, empty space is best discussed later in the context of arguments concerning the void.

Time, similarly, is not an existing reality in which or through which motion occurs. For Aristotle, time was rather "the number of motion with respect to before and after." Time is a number or measure of motion because

[12] Aristotle, *Physics*, IV.1–5.208a27–213a11.

[13] Ibid., IV.1.208b8–27; Aristotle, *De caelo*, IV.3–5.310a14–3313a14; Pierre Duhem, *Theories of Infinity, Place, Time, Void, and the Plurality of Worlds*, ed. and trans. Roger Ariew (Chicago: University of Chicago Press, 1985), pp. 139–268; Edward Grant, "Place and Space in Medieval Physical Thought," in *Motion and Time, Space and Matter: Interrelations in the History of Philosophy and Science*, ed. Peter K. Machamer and Robert G. Turnbull (Columbus: Ohio State University Press, 1976), pp. 137–67, reprinted in Edward Grant, *Studies in Medieval Science and Natural Philosophy* (London: Variorum, 1981); and Edward Grant, "The Concept of *Ubi* in Medieval and Renaissance Discussions of Place," *Manuscripta*, 20 (1976), 71–80, reprinted in Grant, *Studies in Medieval Science and Natural Philosophy*.

it is by time that we measure one motion to be longer or faster than another. Unlike whiteness or heat, which are internal to the body, time is external to the moved body, similar to distance in local motion, which is the other external measure of motion with respect to longer and faster. But whereas distance measures only local motion with respect to here and there (i.e., with respect to place), time measures all motions with respect to before and after, now and then. And just as distance or space is the metric abstraction arising from the extension of bodies, time is the metric abstraction arising from the motion of bodies. Motion is thus prior to and more fundamental than time.[14]

MOTION IN A VOID

Whether there could be motion through a void space and what its characteristics might be were subjects of considerable controversy among medieval commentators on Aristotle's *Physics*. Although most followed Aristotle in asserting that actually existing void spaces are impossible by nature, the general belief that an omnipotent God could create a void provided the occasion for considering hypothetical motion through such a void and for examining the nature of motion in general, the role of the medium, and the relation between moving powers and resistances.

The ancient atomists Democritus and Leucippus had posited the existence of an empty space or void to allow their atoms to move rather than be fixed motionless against each other. The force of Aristotle's arguments in Book IV of *Physics* against the existence of the void was to show that positing a void not only could not serve the purpose intended but would also result in certain absurdities concerning motion. Natural motion, he argued, requires that there be a place in which the body is now located and a place toward which it naturally tends. But if there were a void space, places and directions in it would be undifferentiated, so that there could be neither natural nor violent motion. Nor would there be any reason for a body to come to rest in any particular place, so in a void all bodies would be at rest; or, if they were moved, having no natural tendency or any medium to counteract their motion, they would be moved indefinitely unless stopped by a more powerful body, which Aristotle took to be self-evidently absurd. Again, since according to Aristotle some violent motions depend on the medium (notably projectile motion, discussed later), these also would be impossible in a void. If, as the atomists asserted, the void were the cause of motion by drawing a body into itself, then a body surrounded by a void would tend to be moved equally in all directions; that is, it would not be moved at all. Furthermore, the ratio of

[14] Aristotle, *Physics*, IV.10–14.217b29–224a17. See Duhem, *Theories of Infinity, Place, Time, Void, and the Plurality of Worlds*, pp. 295–363.

speeds of a given body moving through two different resisting media is as the ratio of the rarity of the media, but between a medium however rare and the void there is no ratio, which implies that the speed of motion in a void would be infinite. Finally, since bodies with greater weight divide a medium more readily and thus move more swiftly, in a void all bodies, whatever their weights, will move at equal speeds, which Aristotle took to be impossible.[15] As for interstitial voids – the supposed microscopic voids between the atomists' particles – they are not needed to explain condensation and rarefaction. Like temperature and weight, rarity and density for Aristotle were a kind of qualitative accident changed by alteration, though in this case their intensity is measured by size alone rather than in degrees or pounds. Thus, even by the atomists' own assumptions, the void is neither sufficient nor necessary to explain either local motion or increase and decrease.[16]

Some of Aristotle's later commentators, however, used the hypothetical void to analyze the nature and conditions of motion itself. In particular, Aristotle had argued that, if the void were the sole cause of motion, there could be no ratio between the speed of a motion in a void and that of a motion in a medium. The implications of this – that motion in a void would be instantaneous and consequently that the temporal successiveness of motion depended on a resisting medium – were to have a long history among medieval commentators. Averroes reported that Avempace (Ibn Bajja, d. 1138), perhaps influenced by the sixth-century Greek commentator John Philoponus (d. ca. 570), asserted that motion does not require a resisting medium in order to have a finite speed. As an example, Avempace cited the motions of the celestial spheres, which despite the absence of anything to resist their motion still move with finite speeds. Rather, he argued, the speed of a body is determined by the ratio of the body moved to the cause of its motion, any medium serving only to slow the motion further by subtracting speed. This slowing or retardation, and not the speed itself, is thus proportional to the resistance of the medium.[17]

[15] Aristotle, *Physics*, IV.6–9.213a12–216b21.

[16] Ibid., IV.9.216b22–217b28.

[17] Edward Grant, "Motion in the Void and the Principle of Inertia in the Middle Ages," *Isis*, 55 (1964), 265–92, reprinted in Grant, *Studies in Medieval Science and Natural Philosophy*; James A. Weisheipl, "Motion in a Void: Aquinas and Averroes," in *St. Thomas Aquinas, 1274–1974: Commemorative Studies*, ed. A. A. Mauer, 2 vols. (Toronto: Pontifical Institute of Mediaeval Studies, 1974), vol. 1, pp. 467–88, reprinted in Weisheipl, *Nature and Motion in the Middle Ages*, pp. 121–42. See also Duhem, *Theories of Infinity, Place, Time, Void, and the Plurality of Worlds*, pp. 369–414; Ernest A. Moody, "Ockham and Aegidius of Rome," *Franciscan Studies*, 9 (1949), 417–42, reprinted in Ernest A. Moody, *Studies in Medieval Philosophy, Science, and Logic* (Berkeley: University of California Press, 1975), pp. 161–88 at pp. 166–70; and Ernest A. Moody, "Galileo and Avempace: The Dynamics of the Leaning Tower Experiment," *Journal of the History of Ideas*, 12 (1951), 163–93, 375–422, reprinted in Moody, *Studies in Medieval Philosophy, Science, and Logic*, pp. 203–86 at pp. 226–9.

Averroes himself advanced a different opinion regarding motion in a void. He took Aristotle to require that every body be moved by a mover in constant contact with it (a *motor conjunctus*, or conjoined mover). In the case of living things, the soul, which is the form of a living thing, is the mover of the matter. In nonliving things, the form is also the mover, but it cannot move the matter directly since otherwise there would be no distinction between living and nonliving things. Instead, Averroes argued, the form somehow moves the medium, which in turn moves the matter of the body. Therefore, in the absence of a medium, inanimate natural motion could not occur. But even if it did, such motion would be instantaneous, for matter alone is purely passive and thus can offer no resistance to form, which is pure actuality. Nevertheless, Averroes concluded that, despite the absence of a resisting medium, the motion of the celestial spheres is not instantaneous because the spheres themselves offer the resistance to their movers necessary for finite speeds.[18]

Although Averroes's views on motion in a void exerted considerable influence through the high Middle Ages, on this point as on many others Thomas Aquinas offered an important alternative to the Averroist interpretation. According to Aquinas, motion in a void would not be instantaneous since a body, even with all its other forms removed (including heaviness and lightness), simply by virtue of being extended, has a natural inertness (*inertia*) to being moved. Note that this inertness does not arise from heaviness or some other property of the body since these have been explicitly removed from the body – still less does it arise from mass or quantity of matter in Newton's sense. Rather, it arises from the purely spatial extension of the body. Similarly, the magnitude through which a body is moved also has spatial extension, and since its parts are in sequence, so are the parts of the motion itself. Thus, for Aquinas, motion takes a finite time even in the absence of a medium and all other resistance, which would serve only to subtract from the body's natural speed. This argument came to be known as the "incompossibility of termini" (*incompossibilitas terminorum*) or "separation of termini" (*distantia terminorum*) since the mere separation of places (*termini*) and the impossibility of the same body's occupying them simultaneously were seen to be sufficient causes of the finite successiveness of motion. Nevertheless, most natural philosophers in the fourteenth century, including Aegidius of Rome, Walter Burley, and John of Jandun, followed the opinion of Averroes against Avempace. William of Ockham, however, saw no contradiction between the separation of termini as the cause of the successiveness of motion and Avempace's view that speeds decrease in proportion to the resistance of the medium. Ockham's resolution of the problem in effect distinguished between motion considered purely in terms of distance and time, and motion considered in relation to

[18] Weisheipl, "Motion in a Void," pp. 131–4; and Moody, "Galileo and Avempace," pp. 229–41.

its causes – a distinction that would be made explicit by later scholars at Oxford.[19]

Natural philosophers sometimes turned Aristotle's arguments concerning the void on their head, so that the absurd consequences of positing a void were simply accepted as hypothetically true and then analyzed and explained. In his *Treatise on the Ratios of Speeds in Motions* (*Tractatus de proportionibus velocitatum in motibus*) of 1328, the Oxford theologian Thomas Bradwardine (d. 1349) included a theorem stating that in a void all mixed bodies of similar composition would in fact move at equal speeds whatever their sizes, which Aristotle had deemed absurd. A mixed body was one compounded of several or all of the elements; because the elements are never actually found pure and unmixed, all actual physical bodies are mixed bodies. Since each of the elements has a natural tendency to be moved up or down, a mixed body, according to Aristotle in *De caelo*, will be moved with the tendency of the predominant element.[20] In refining these suggestions of Aristotle, medieval commentators usually identified the predominant element as the motive power and the elements with the contrary tendency as the resistive power. Depending on where the body was, one element of the body could be indifferent to motion and thus would contribute to neither motive nor resistive power. For example, a body located in air and consisting of three parts of fire and two parts each of earth, air, and water has in air a motive power downward since the downward power of the earth and water combined overcomes the upward power of the fire, whereas the air in its own element tends neither up nor down. In water, however, the body has an upward tendency since the upward motive power of the fire and air together exceeds the downward power of the earth, whereas the water this time has no tendency up or down.

There was considerable debate over whether the resulting speed depended on the excess of motive over resistive power (found by subtracting the resistive power from the motive) or on the ratio of the motive to the resistive power. Bradwardine, as will be shown at length in the next section, asserted that speeds follow the ratio of motive to resistive powers. And since mixed bodies of the same composition (that is, with the same ratio of elements) have the same ratio of motive to resistive powers, their speeds will be the same in a void. Bradwardine acknowledged that this conclusion was not at all obvious. He noted that it seems to contradict the common Aristotelian opinion that bodies move naturally at speeds in proportion to their magnitudes. But he explained that because the intensities of the motive forces are in the same ratio as the intensity of the resistances, they produce equal *intensities* of

[19] Weisheipl, "Motion in a Void," pp. 134–42; Grant, "Motion in the Void and the Principle of Inertia in the Middle Ages," pp. 268–72; and Moody, "Galileo and Avempace," pp. 241–56.

[20] Aristotle, *De caelo*, I.2.268b30–269a6.

motion (that is, equal speeds) despite the difference in their absolute sizes or *extensions*.

This distinction between the intensity and extension of motion would become a significant part of the latitude of forms subsequently pursued at Oxford and Paris in the next generation. Albert of Saxony (ca. 1316–1390) arrived at the same conclusion and went on to contrive a number of paradoxical results concerning the speeds of mixed and pure elemental bodies moving in various media and the void, including a case in which natural motion would be slower at the end than at the beginning. As for a hypothetical body composed of a single element moving in a void, Bradwardine had declined to assign it any definite speed, whereas Albert asserted that its speed would be infinite.[21]

BRADWARDINE'S RULE

In Book VII of *Physics*, Aristotle had treated in general the relation between powers, moved bodies, distance, and time, but his suggestions there were sufficiently ambiguous to give rise to considerable discussion and disagreement among his medieval commentators. The most successful theory, as well as the most mathematically sophisticated, was proposed by Thomas Bradwardine in his *Treatise on the Ratios of Speeds in Motions*. In this tour de force of medieval natural philosophy, Bradwardine devised a single simple rule to govern the relationship between moving and resisting powers and speeds that was both a brilliant application of mathematics to motion and also a tolerable interpretation of Aristotle's text.

In what is known as the ship-haulers argument, Aristotle had argued that if a certain power (e.g., a certain number of men) can move a certain body (e.g., a ship) a certain distance in a certain time, that same power can also move half that body the same distance in half the time or twice the distance in the same time. Similarly, twice that power can move the same body twice the distance in the same time or the same distance in half the time. But half the power does not necessarily move the same body half the distance in the same time or the same distance in twice the time, for the power may be too weak to move the body at all.[22]

[21] H. Lamar Crosby, Jr., ed. and trans., *Thomas of Bradwardine: His Tractatus de Proportionibus, III.2*, Theorem XII (Madison: University of Wisconsin Press, 1961), pp. 116–17; and Edward Grant, "Bradwardine and Galileo: Equality of Velocities in the Void," *Archive for the History of Exact Sciences*, 2 (1965), 344–64, reprinted in Grant, *Studies in Medieval Science and Natural Philosophy*, pp. 345–55.

[22] Aristotle, *Physics*, VII.5.249b27–250b7. See Israel E. Drabkin, "Notes on the Laws of Motion in Aristotle," *American Journal of Philology*, 59 (1938), 60–84.

It was not Aristotle's purpose here to state a precise quantitative rule governing powers and motions. Rather, this argument was one of the preliminaries leading to his proof in Book VIII of *Physics* that there must be a first mover and a first mobile body. Medieval commentators, however, considered the ship-haulers argument to be his definitive statement of the relationship between movers, resistances, and speeds, and they tried to put his rule into more definite form. They identified powers and moved bodies as moving powers and resisting powers, which, since both were powers, could then be put into ratios with each other. They thought of speed not in the modern sense as a ratio of distance to time (since such dissimilar quantities cannot form ratios) but as the intensity of motion, measured in degrees. (A motion with two degrees of speed, for example, is twice as fast as a motion with one degree of speed and will therefore cover twice the distance in the same time or the same distance in half the time.) Several interpretations of the relationship between motive and resisting powers and the resulting speeds had been proposed by Aristotle's ancient and medieval commentators in order to reconcile the rules of motion in Book VII of *Physics* with other passages and with common observation. The goal was to come up with a single general rule that expressed the relationship between moving and resisting powers and speed and at the same time precluded motion when the moving power is less than or equal to the resisting power, a goal that Bradwardine's rule achieved brilliantly.

Bradwardine wrote his *Treatise on the Ratios of Speeds in Motions* in the form of a deductive argument, beginning with definitions and suppositions and proceeding to theorems and proofs. In the first of its four chapters, he laid the necessary mathematical groundwork – the medieval theory of ratios and proportions, which until then had been applied mainly to music theory. The crux of Bradwardine's treatment is what it means for a ratio to be a *multiple* of – or, in general, greater than – another ratio. A ratio is a *multiple* (e.g., *double* or *triple*, in a technical sense that applies only to ratios) of another ratio when it can be compounded from that smaller ratio. For example, the ratio 4:1 is *double* 2:1 since 2:1 compounded twice with itself or *doubled* – (2:1)(2:1), or $(2:1)^2$ – yields the ratio 4:1; similarly, 8:1 is *triple* 2:1. And in general this is what Bradwardine meant when he said that one ratio is *greater than* another – that the larger is compounded from the smaller.[23]

Before using the theory of compounded ratios in his own rule, Bradwardine considered and rejected four alternative opinions on the relationship between powers, resistances, and speeds. The first is that speeds follow the excesses of motive powers over resistances ($V \propto [M-R]$, where $V =$ speed, $M =$ motive power, and $R =$ resistance), the opinion attributed to Avempace and others and discussed previously in connection with motion in a void. The second opinion, which Bradwardine attributed to Averroes, is that

[23] Crosby, *Thomas of Bradwardine*, I.3, pp. 76–81.

speeds follow the ratio of the excesses of the motive over the resisting powers to the resisting powers ($V \propto [M-R]/R$). The third opinion is that the speeds follow the inverse of the resistances when the moving powers are the same ($V \propto 1/R$ when M is constant) and follow the moving powers when the resistances are the same ($V \propto M$ when R is constant), which was the usual interpretation of Aristotle's rule. The fourth opinion is that speeds do not follow any ratio because motive and resistive powers are quantities of different species and so cannot form ratios with each other. Bradwardine rejected all of these because they are either inconsistent with Aristotle's opinion or yield results contrary to common experience.[24]

Bradwardine's own rule is that the ratio of speeds follows the ratios of motive to resistive powers. This deceptively simple formulation appears at first to be no different from the third rejected opinion until one understands it in the light of medieval ratio theory just explained. Speed is tripled, for example, when the ratio of motive to resistive power is *tripled* – in the technical sense that applies to ratios. For example, if a motive power of 2 moves a resistance of 1 at a certain speed, a motive power of 8 will move the same resistance with a speed three times the first speed since the ratio 8:1 is *triple* the ratio 2:1; that is, 8:1 is compounded of 2:1 with itself three times (i.e., $8{:}1 = (2{:}1)(2{:}1)(2{:}1)$ or $(2{:}1)^3$). And, in general, speeds are doubled or tripled or halved as the ratios between motive and resistive powers are *doubled* or *tripled* or *halved* in the technical sense that applies to ratios.[25]

Furthermore, Bradwardine's rule precludes the possibility of motion when the motive power becomes equal to or less than the resisting power because no ratio of greater inequality of motive to resisting power (e.g., 8:1) can be compounded of ratios of equality (e.g., 1:1 or 2:2) or of lesser inequality (e.g., 1:2), for no matter how many times one compounds ratios like 1:1 or 1:2 with themselves, one will never arrive at a ratio like 8:1. This means effectively that speed approaches zero as the ratio of motive to resisting power approaches equality; when the motive power is less than the resisting power, there will be no speed at all.[26] Thus, by applying medieval ratio theory to a controversial passage in Aristotle's *Physics*, Bradwardine devised a simple, definite, and sophisticated mathematical rule for the relation between speeds, powers, and resistances.

The rest of the *Treatise* consists of a series of theorems in which Bradwardine applied this rule to various combinations of motive and resistive powers. Where the ratios were *multiples*, he could determine the speeds as multiples, but in the many more cases where they were not *multiples*, he was content with proving only whether the resulting speed would be greater than, less than, or equal to some multiple of the original speed. He did not resort to

[24] Ibid., II, pp. 86–111.

[25] Ibid., III.1, pp. 110–13; and Anneliese Maier, "The Concept of the Function in Fourteenth-Century Physics," in Sargent, *On the Threshold of Exact Science*, pp. 61–75.

[26] Crosby, *Thomas of Bradwardine*, II, pp. 81–6, 114–15.

the actual measurement of powers or speeds to confirm his results, beyond his earlier appeals to common experience to disprove the alternative rules. Nevertheless, his rule led him to the remarkable conclusion, mentioned earlier, that all mixed heavy bodies of similar composition will move naturally at equal speeds in a vacuum.[27]

Bradwardine's rule was widely accepted in the fourteenth century, first among his contemporaries at Oxford, where Richard Swineshead and John Dumbleton applied it to solving sophisms, the logical and physical puzzles that were just beginning to assume an important place in the undergraduate arts curriculum (See Ashworth, Chapter 22, this volume.). Soon thereafter it appeared at Paris, in the works of Jean Buridan and Albert of Saxony, and, by mid-century, at Padua and elsewhere. It circulated not only in copies of Bradwardine's own work but even more widely in an abbreviated version dating probably from the mid-fourteenth century.[28]

In one of the more interesting applications of Bradwardine's rule to a physical problem, Richard Swineshead (fl. 1340–55), known to posterity as "the Calculator" from the title of his influential *Book of Calculations*, set out to determine whether a natural body acts as a unified whole or as the sum of its parts. He imagined a long, uniform, heavy body falling in a vacuum (to eliminate the complication of a resisting medium) down a tunnel through the center of the earth. The motive power is represented by the length of the bar above the center and still falling, and the resisting power is represented by the length of the bar that has already fallen past the center and so is now rising on the other side. Using Bradwardine's rule – that the speed of the bar follows the ratio of motive to resisting power – he examined each successive speed as the distance between the center of the bar and the center of the world is successively halved. He found that the speed of the bar decreases in such a way that its center will never reach the center of the world in any finite time. But Swineshead in the end rejected this conclusion because of the unacceptable consequence that the nature of the whole bar as a heavy body would be thwarted since its center could never coincide with its natural goal, the center of the world. Instead, he argued that all the parts of the body would assist the whole in achieving its goal. Despite the subtlety of the analysis, then, for Swineshead the physical argument trumped the mathematical.[29]

Elsewhere in his *Book of Calculations*, Swineshead applied Bradwardine's rule to the analysis of motions through variously resisting media, devising rules to compare not only speeds but the rate of change of speeds when either

[27] Ibid., III.1, pp. 112–17; the conclusion mentioned is Theorem XII, pp. 116–17.

[28] Clagett, *Science of Mechanics in the Middle Ages*, pp. 421–503, 629–52.

[29] Michael A. Hoskin and A. G. Molland, "Swineshead on Falling Bodies: An Example of Fourteenth-Century Physics," *British Journal for the History of Science*, 3 (1966), 150–82; A. G. Molland, "Richard Swineshead and Continuously Varying Quantities," in *Acts of the Twelfth International Congress on the History of Science, Paris, 1968*, 12 vols. in 15 (Paris: Albert Blanchard, 1971), vol. 4, pp. 127–30; and Molland, Chapter 21, this volume.

the moving or the resisting power is continuously increased or decreased with time. He also considered more complex cases, such as a body moved by a constant power through a medium the resistance of which increases uniformly with distance (rather than with time). Since the speed of the body at any instant depends on where it is in the medium but where it is in the medium depends in turn on its speed and the time it has traveled, to solve this problem Swineshead effectively had to reduce the distance dependency of speed to time dependency.[30]

The only other significant extension of Bradwardine's rule – of more interest to the history of mathematics than of the science of motion – was by Nicole Oresme. As mentioned earlier, where the ratios were not simple *multiples* of other ratios, Bradwardine could calculate only whether the resulting speeds were larger or smaller than some multiple. Oresme, however, generalized what he called the ratio of ratios to include improper fractional and even irrational ratios of ratios, which extended the mathematical rigor and generality of Bradwardine's rule but without changing its basic application to motion.[31]

FALLING BODIES AND PROJECTILES

Two of the most controversial problems in the medieval science of motion concerned the acceleration of falling bodies and the continued motion of projectiles. The ancient commentators had already criticized Aristotle's treatments, and medieval commentators in turn tried to solve these problems in ways that were consistent both with general Aristotelian principles and with everyday experience. In the effort to explicate Aristotle, however, they devised solutions that went far beyond what could reasonably be attributed to Aristotle, producing some of the most interesting innovations in medieval physical thought. And, in the process, Aristotle's distinction between natural and violent motions tended to become blurred, implicitly calling into question a fundamental tenet of Aristotelian natural philosophy.

Falling bodies and projectiles came up in the course of Aristotle's arguments for a first or prime mover in Book VIII of *Physics*. The main premise of the argument was that everything that is moved is moved by another. To establish this premise, Aristotle distinguished three classes of motion according to the mover's relationship to the thing moved: the motion of living (self-moving) things, forced (compulsory or violent) motion, and natural (or spontaneous) motion. He then showed that in each case the moved body is distinct from its mover. This distinction is clearest in the case of forced or

[30] John E. Murdoch and Edith Dudley Sylla, "Swineshead, Richard," *Dictionary of Scientific Biography*, XIII, 184–213, especially pp. 201–4.

[31] Nicole Oresme, *De proportionibus proportionum and Ad pauca respicientes*, ed. and trans. Edward Grant (Madison: University of Wisconsin Press, 1966), especially pp. 24–36.

violent motion, for there the cause of the motion is obviously different from the body moved. The horse moves the cart, and the bowstring moves the arrow. Where motion is imposed on a body from without, the mover must be in continuous contact with the body in order to endow it with motion. The difficulty with violent or forced motion was not in distinguishing the mover from the moved but rather in determining why the motion continues once the mover is no longer in contact with the moved body. This is the problem of projectile motion, which will be treated later in the chapter.

In the case of living things, since they are self-moving, there would seem to be no distinction between mover and moved. But here Aristotle distinguished self-movements proper – voluntary movements that arise from appetite – from involuntary movements such as growth, nutrition, generation, heartbeat, and the like that arise from the nature of the animal as a certain kind of living thing. Properly speaking, animals move themselves by walking, running, crawling, swimming, flying, and such, motions the efficient cause of which is the animal itself but understood in the sense of "part moving part." That is, in every case of an animal's moving itself, the motion is caused by one part of the animal first moving another part and then being moved in turn. In this way, Aristotle preserved the distinction between mover and moved in animals that move themselves and thus preserved the generality of the dictum that everything that is moved is moved by another.

But the most difficult cases, Aristotle admitted, are the involuntary movements of living things and the natural motions of inanimate bodies. These movements arise from the nature of the body, nature being the source of motion in those things that tend to move by virtue of what they are. In such cases, the immediate efficient cause of the motion is the generator of the body itself; that is, whatever brought the body into being as a heavy – or as a living – thing in the first place. Once a body has been endowed by its generator with a certain nature, it acts spontaneously in accordance with that nature, no further efficient cause being necessary. A heavy body, for example, will spontaneously fall by nature unless it is impeded by some other body. Thus the only other efficient cause that Aristotle admitted for natural motion is the cause that removes an impediment, although this is only an incidental rather than an essential cause of motion. For Aristotle, inanimate elemental bodies are not to be thought of as self-moving. Since they are homogeneous (that is, indefinitely divisible into parts of similar composition), there is in them no distinction of parts and therefore no first part that could move the other parts. And since they are not moved part by part, they are not self-moving.[32]

The nature of a body, which is the source and principle of its motion, is expressed in its formal and final causes; for example, the form of a heavy

[32] Aristotle, *Physics*, VII.1.241b34–242a49 and VIII.4.254b7–256a3; and Weisheipl, "The Principle *Omne quod movetur ab alio movetur* in Medieval Physics," *Isis*, 56 (1965), 26–45.

body makes it heavy and causes it to seek the center of the cosmos, its goal or final cause. For this reason, Avicenna identified the form of a natural body or its natural goal as efficient causes of its natural motion. Averroes, although allowing that the generator gives a body its form and all of its characteristic activities, including local motion, nevertheless argued that Aristotle required a mover conjoined to the body at all times, even in natural motion. Since only in living things can the form be the direct mover of the body, Averroes suggested that in inanimate natural bodies the form first moves the medium, which in turn moves the body itself, like a man rowing a boat. Thus, for Averroes the medium plays the same role in natural motion as it did for Aristotle in the continuation of forced, projectile motion.

Although both Albertus Magnus and Thomas Aquinas argued against Averroes's interpretation of Aristotle, the tendency to look for an efficient cause of natural motion within the moved body proved irresistible to many subsequent thinkers. One notable exception was Duns Scotus, who denied that everything moved is moved by another, asserting instead that all natural and voluntary motions are in fact self-motions.[33] However, many later commentators tended, like Averroes, to look within the naturally moving body itself for an efficient cause of its motion, similar to the cause of violent motion, especially of projectiles. Consequently, they posited a cause of forced motion, often called *impetus*, that inhered almost like an artificial, contrary nature in a moving projectile. Impetus was also sometimes invoked to explain the acceleration of freely falling heavy bodies. Thus, in the fourteenth century, the causes of natural and violent motion tended to be assimilated to each other; by the seventeenth century, the distinction between natural and violent motions would break down entirely in the physics of Galileo and Descartes. We shall return to the cause of the acceleration of falling bodies after discussing impetus theory.

PROJECTILE MOTION AND THE THEORY OF IMPETUS

In the course of his argument for a first motion and a first mover in Book VIII of *Physics*, Aristotle considered the possibility that the first motion (which he identified with the rotation of the outermost cosmic sphere) could be like that of a projectile in that it might continue not through the continued action of a first mover but rather as a stone continues to be moved through

[33] James A. Weisheipl, "The Specter of *Motor coniunctus* in Medieval Physics," in *Studi sul XIV secolo in memoria di Anneliese Maier*, ed. A. Maierù and A. P. Bagliani (Rome: Edizioni di Storia e Letteratura, 1981), pp. 81–104, reprinted in Weisheipl, *Nature and Motion in the Middle Ages*, pp. 99–120; and James A. Weisheipl, "Aristotle's Concept of Nature: Avicenna and Aquinas," in *Approaches to Nature in the Middle Ages: Papers of the Tenth Annual Conference of the Center for Medieval and Early Renaissance Studies*, ed. Lawrence D. Roberts (Medieval and Renaissance Texts and Studies, 16) (Binghamton, N.Y.: Center for Medieval and Early Renaissance Studies, 1982), pp. 137–60.

the air after it has left the hand or the sling that has thrown it. He rejected this possibility, however, by asserting a theory of projectile motion that would later be accepted by few of even his most loyal followers. According to Aristotle, the thrower imparts not only motion to the projectile and to the medium surrounding it but also a certain power of moving to the medium alone, either air or water, which has a natural propensity to receive such a power of moving. Each part of the medium so affected thus becomes a mover of the projectile by passing both motion and the power of moving to its adjacent parts. In passing from one part of the medium to the next and then to the projectile, both the motion and the power of moving are weakened, in part at least by the natural tendency of the projectile as a heavy or light body to be moved down or up. Eventually, the power of moving is so weak that the last part of the medium can pass on motion but not the power of moving. When this last motion of the medium ceases, so does the forced motion of the projectile.[34]

In Aristotle's defense, it must be said that his main purpose was not to explain projectile motion but rather to show that it was an unsuitable candidate for the first motion, for he had shown earlier that the first motion must be absolutely continuous, which in his theory the motion of a projectile is not since it is caused by a succession of different movers (i.e., the different parts of the medium). Nevertheless, his views on projectile motion received devastating criticism from John Philoponus, who in the sixth century first advanced most of the arguments that would be leveled against it over the next millennium. In general, Philoponus argued that a medium can act only to resist the motion of a projectile; what keeps it moving is an "incorporeal motive force" received from the projector. Although Philoponus's criticisms and alternative theory were not known directly in the Latin West, they probably influenced several Islamic philosophers, including Avicenna, who argued that projectiles receive from the projector an inclination – in Arabic, *mayl* – that inheres in the body and is not a motive force but only its instrument. But since these passages of Avicenna were not fully translated into Latin, the theory of *mayl* had little influence in the Latin Middle Ages, though it formed part of a parallel tradition in Arabic that lasted into the seventeenth century.[35]

In the Latin West, the first alternative to Aristotle's theory of projectile motion appeared, surprisingly, in the context of sacramental theology. In his commentary on Peter Lombard's *Sentences*, the Franciscan theologian Peter John Olivi (d. 1298) discussed how the sacraments could continue to be effective long after they had been instituted by Christ. By way of analogy, Olivi described a theory of projectile motion wherein the projector imparts to the projectile a similitude or *species* of itself as mover. Olivi was apparently

[34] Aristotle, *Physics*, VIII.10.266b27–267a15.
[35] Clagett, *Science of Mechanics in the Middle Ages*, pp. 505–14.

drawing on the species theory of Roger Bacon (ca. 1220–ca. 1292), which explained all causation by means of species or images of a cause imposed on other things.[36] In a later work, however, Olivi rejected this theory of projectile motion on the grounds that, since motion is a real mode of being in itself that needs nothing to sustain it, the continuation of a projectile needs no cause besides the original projector. This later position is identical to that taken by William of Ockham, though for exactly opposite reasons. Ockham not only rejected all species as unnecessary in the explanation of causes and their effects, but he also claimed that local motion was nothing over and above a body and the places it occupies successively without stopping in any. Since the continued motion of a projectile is not a new effect, it has no need of a cause at all. Around the same time, another theologian, Franciscus de Marchia, again drew the analogy between sacramental causality and projectile motion. According to Franciscus, in his lectures on the *Sentences* given at Paris in 1319–20, the mover gives to the projectile – rather than to the medium, as Aristotle thought – a certain moving power that remains for a time in the projectile and causes its motion to continue.[37]

Whether informed by these theological treatments or not, the philosopher Jean Buridan, twice rector of the University of Paris between 1328 and 1340, presented the canonical version of what he called *impetus* in his questions on Aristotle's *Physics* and *De caelo*. But first he mustered a decisive set of arguments against the view that the medium is somehow responsible for the continuing motion of projectiles. First, a top or a smith's wheel will continue to rotate in place long after it is pushed, despite the fact that air cannot get behind it, and continues to rotate even when one obstructs the air by placing a cloth next to its edge. Again, a lance pointed at both ends can be thrown as far as one pointed only in front, though the point behind presents less area for the air to push against. Moreover, air is so easily divided that it is difficult to see how it could move a thousand-pound stone thrown from a siege engine. Furthermore, ships will continue to move without there being any wind detected behind them, and no matter how hard one beats the air behind a stone, little motion is produced compared with throwing it. And finally, if the air moved projectiles, then a feather could be thrown farther than a stone since it is more easily moved by air, but the contrary is observed. Therefore, Buridan concluded, a projectile is not moved by the air but by a certain *impetus* or motive force (*vis motiva*) impressed on it by the mover in proportion to the mover's speed and to the amount of prime matter in the projectile. The air, along with the natural tendency of the heavy body to move downward, serves only to diminish the impetus by resisting the motion, so that the forced motion becomes slower and slower until it is completely overcome. This is why larger and denser bodies can be thrown

[36] See Lindberg and Tachau, Chapter 20, this volume.
[37] Ibid.

farther, for a greater impetus can be impressed on them by virtue of their greater quantity of prime matter, and why a greater resistance of air and natural tendency is necessary to overcome their motion. Buridan took pains to point out that impetus is not the same as the motion it produces, for motion is present only successively, one part after another, whereas impetus is a quality or accident present all at once in its subject. Impetus is thus a real motive power in the body it moves, impressed on it by the mover and inhering in it in the same way that a magnet impresses a quality on a piece of iron that causes it to be moved.[38] Using this idea of impetus, Buridan went on to explain a variety of physical phenomena, including the rebound of bodies from obstacles, the vibration of strings, the swinging of bells, and, most notably, the motion of the celestial spheres and the acceleration of falling bodies, to which we now turn.

ACCELERATION OF FALLING BODIES

According to Aristotle, natural motion (such as the descent of a heavy body) is faster at its end than at its beginning. He implied in *Physics* and *De caelo* that the reason for this in the case of falling bodies is that the closer a heavy body comes to the center of the cosmos (its natural place), the heavier it becomes and the faster it will fall.[39] Aristotle's ancient commentators gave several alternative explanations of the acceleration of falling bodies, attributing it either to the medium through which they fall or to some power temporarily resident in the body itself. Simplicius (sixth century), in his commentary on *De caelo*, recorded that, according to the astronomer Hipparchus, whatever power had lifted or sustained the heavy body continues to act on it as it falls, resulting in an increase in speed as this power gradually wears off. Others (Simplicius did not identify them) suggested that a lesser quantity of medium beneath a falling body offers less support, so that the lower the body gets, the faster it goes. In the end, however, Simplicius agreed with Aristotle, although he acknowledged that Aristotle's explanation implied that a body should weigh more when it is in a lower place unless perhaps the difference is insensible.

Philoponus, for his part, implicitly denied Aristotle's explanation of the acceleration of falling bodies when he argued in his commentary on *Physics* that two different weights fall with almost the same speed and not with speeds proportional to their weights. One medieval Arabic commentator, Abu ʿl-Barakat (d. ca. 1164), under the influence of Philoponus's explanation of projectile motion, argued that a projectile, after it has ceased moving upward

[38] Clagett, *Science of Mechanics in the Middle Ages*, pp. 532–40; and Maier, "The Significance of the Theory of Impetus for Scholastic Natural Philosophy," in Sargent, *On the Threshold of Exact Science*, pp. 76–102 at pp. 86–7.

[39] Aristotle, *De caelo*, I.8.277a27–33; and Aristotle, *Physics*, IV.8.216a13–16.

and begins to fall, still retains some of its upward, violent inclination, or *mayl*, as it begins to fall, and because this violent *mayl* continues to weaken while the natural *mayl* increases as the body falls, the body speeds up. Averroes, in contrast, although agreeing with Aristotle's explanation, added that a falling body also speeds up because as it falls it heats the air through which it passes, which then becomes more rarefied and therefore offers less resistance.[40]

Drawing on Averroes and perhaps on Simplicius, whose commentary on *De caelo* had been translated by William Moerbeke in 1271, Jean Buridan offered a number of arguments against both Aristotle and those who attributed the acceleration of falling bodies to the medium. Against Averroes, he noted that bodies do not fall faster in the summer, when the air is hot, than in the winter, and that a man does not feel the air heat up as he moves his hand through it. Against Aristotle, he argued, as Philoponus had, that a stone is not lighter the higher up it is. When dropped from a high place, it moves much more quickly through the last foot of its fall than a stone dropped to the same level from the height of a foot; nor does a stone dropped from a high place through some distance fall faster than a stone dropped from a low place through an equal distance. For similar reasons, he rejected the opinion that the greater the amount of air beneath a falling body, the greater the resistance, suggesting instead that the air higher up should be rarer and thus less resistant. He also suggested that air closer to the ground should in fact offer more resistance to a falling body, not less, because there is less room for it to be pushed out of the way. Finally, he dismissed the suggestion that acceleration is caused by an insensible difference in weight between a body in a high place and one in a low place on the grounds that since the differences in speed and impact are very sensible, they cannot result from an insensible difference in weight.[41]

For all of these reasons, Buridan concluded that both the weight of the stone and the resistance of the medium are constant throughout the fall and thus do not contribute to acceleration. He suggested instead that, as a heavy body falls, it is moved not only by its gravity or weight (which remains constant and thus by itself would produce only a constant speed) but also by a certain impetus that the body acquires from its motion. At first, the body is moved only by its gravity, so it moves only slowly; then it is moved both by its gravity and by the impetus acquired from its first motion, so it moves more swiftly; and so on, its speed continually increasing as more and more impetus is added to its natural gravity. Buridan noted that this impetus could properly be called "accidental gravity," which placed him in agreement with both Aristotle and Averroes that increasing gravity is the cause of acceleration. Having rebutted earlier explanations with reasons drawn from a careful analysis of the observed behavior of falling bodies,

40 Clagett, *Science of Mechanics in the Middle Ages*, pp. 541–7.

41 Jean Buridan, *Questions of the Four Books on the Heavens* II, qu. 12, trans. in Clagett, *Science of Mechanics in the Middle Ages*, pp. 557–60.

Buridan thus applied his theory of impetus, originally devised to explain the continued violent motion of projectiles, to explain the acceleration of naturally falling bodies.[42]

Buridan's use of impetus theory to explain the acceleration of falling bodies was widely accepted in the late Middle Ages, by Albert of Saxony and Nicole Oresme, among many others. Albert applied it to a body that falls through a hole passing through the center of the earth, suggesting that its impetus would carry it past the center of the earth until overcome by the body's natural tendency; it would then again descend to the center and continue to oscillate about the center until its impetus was entirely spent. In adopting the impetus theory of falling bodies, Oresme introduced a modification of his own. According to him, impetus arises not from the speed of the falling body but from its acceleration; this impetus in turn causes more acceleration, and so on. He also repeated the now familiar example of a body falling through the center of the earth, which he compared to the swinging of a pendulum, in order to argue that impetus cannot properly be weight or gravity since in these cases it causes heavy bodies to rise, whereas weight can cause them only to fall. All motions, Oresme concluded, whether natural or violent, possess impetus – with the notable exception of the celestial movements.[43]

Compared with the cause of the acceleration of falling bodies, the mathematical rule that governed their actual acceleration received almost no attention in the Middle Ages. In general, medieval natural philosophers tended to assume that the speed of falling bodies increases with the distance and the time of fall, without distinguishing the two and without specifying the exact relationship. Nicole Oresme, however, suggested several alternatives for the rule governing the increase of speed of falling bodies, including the correct one – that equal speeds are acquired in equal times – though he did so only to determine whether the speed would increase indefinitely or approach some finite limit. Besides Oresme, only one other late scholastic philosopher asserted what is now known to be the correct rule to describe the acceleration of falling bodies: the sixteenth-century Spaniard Domingo de Soto, who applied to falling bodies the mean-speed theorem for uniformly accelerated motions, described in the next section.[44]

THE OXFORD CALCULATORS AND THE MEAN-SPEED THEOREM

In the fourteenth century, motion and change provided material for the logical and physical puzzles known as *sophismata*, which came to assume a

[42] Ibid., pp. 560–4.
[43] Ibid., pp. 552–3, 565–71.
[44] Ibid., pp. 541–56; and William A. Wallace, "The Enigma of Domingo de Soto: *Uniformiter difformis* and Falling Bodies in Late Medieval Physics," *Isis*, 59 (1968), 384–401.

significant place in the arts curriculum of the medieval university. *Sophismata* gave rise to an extensive body of literature in the form of handbooks for undergraduate students, in which the quantitative treatment of qualities and the comparison of speeds of motion resulted in a remarkable series of theoretical insights, including the idea of instantaneous velocity, the mean-speed theorem, the distance rule for accelerated motions, and the graphical representation of qualities and motions.[45] In this context, a distinction was drawn between considering the speed of motion with respect to its causes (that is, with respect to its powers and resistances) – as in Bradwardine's rule and impetus theory – and considering it with respect to its effects (that is, with respect to distances and times). This distinction is roughly equivalent to the modern distinction between dynamics and kinematics.

For motion considered with respect to its effects, Aristotle was again the main starting point; in Books VI and VII of *Physics*, he had discussed the comparison of speeds and the continuity and infinity of magnitude, time, and motion, partly to refute Zeno's paradoxes of motion but ultimately to show in Book VIII that the only possible infinite motion is the rotation of a sphere moved by an immaterial mover. In Book VI, he had defined the faster as what traverses a greater magnitude in an equal time or traverses an equal or greater magnitude in less time.[46] Then, in Book VII, he asked whether every change or motion is comparable with every other change or motion. Clearly, a change of place cannot be directly compared with an alteration, for although their times may be compared, a distance cannot be compared with a quality. Similarly, circular motions cannot be compared with straight ones because, for Aristotle, circular and straight were two different species in the genus of line, and since they are different kinds of things, they cannot be compared with each other. Strictly speaking, then, the speeds of circular and straight motions were for Aristotle incomparable.[47]

Apart from Aristotle, the other possible inspiration for the earliest medieval attempts to compare motions was *On the Motion of the Sphere* of Autolycus of Pitane (fl. 310 B.C.). Translated into Latin several times in the twelfth and thirteenth centuries and circulated widely with astronomical and other mathematical texts, *On the Motion of the Sphere* may have been known to Gerard of Brussels, who wrote his *Book on Motion* sometime between 1187 and 1260. Drawing on Archimedes' *On the Measure of the Circle* and a derivative of his *Sphere and Cylinder*, Gerard, like Autolycus, tried to find the point on a rotating line (for instance) such that, if the whole line were moved uniformly and rectilinearly at the speed of this point, its motion would be equivalent to its motion of rotation; that is, it would sweep out

45 Edith Dudley Sylla, "Science for Undergraduates in Medieval Universities," in *Science and Technology in Medieval Society*, ed. Pamela O. Long (Annals of the New York Academy of Sciences, 441) (New York: New York Academy of Sciences, 1985), pp. 171–86.

46 Aristotle, *Physics*, VI.2.232a23–27.

47 Ibid., VII.4.248a10–249b26.

an equal area in the same time. The point he chose for a rotating line was its midpoint, and he applied the same technique to the rotation of surfaces and volumes.[48] In a similar way, Thomas Bradwardine, in the fourth chapter of his *Treatise on the Ratios of Speeds in Motions*, reduced circular motions to their rectilinear equivalents, except that for him the speed of a rotating line was to be measured not by its midpoint but by its fastest point.[49] The solution to *sophismata* that involved the comparison of speeds of rotating bodies often hinged on whether the overall speed of a rotating body should be measured by its midpoint or by its fastest point – in either case, Aristotle's reservations concerning the comparison of rectilinear and circular motions came to be effectively ignored.

A number of scholars at Oxford in the 1330s and 1340s – notably William Heytesbury, Richard Swineshead, and John Dumbleton – avidly took up the study of motion with respect to its effects, developing quasi-mathematical techniques, known as calculations (*calculationes*), to deal with it and its associated problems. Since several of these masters, including Bradwardine, were associated with Merton College, Oxford, they are often referred to as the Merton school or the Mertonians, though the more inclusive "Oxford calculators" is preferable.[50] These scholars considered local motion within the more general context of what they called the intension and remission of forms – how formal qualities and properties in general are intensified and diminished and how they are to be compared as they change. Duns Scotus had suggested that a body became whiter, for example, not by exchanging its existing form of whiteness for an entirely new form of a higher degree or intensity but by an addition of whiteness to the existing form, resulting in a higher degree of whiteness. In this way, qualitative forms were quantified and their quantities expressed in degrees or intensity. Since the range of intensity was called "latitude," the theory concerning the quantity of qualities came to be called the "latitude of forms."

The mathematical techniques developed in connection with the latitude of forms were applied to a variety of natural phenomena. For example, the hotness of a body can be measured in two ways – by its intensity (what we call its temperature) and by its extension (the total quantity of heat in the body, which we measure in calories). Thus a one-pound and a two-pound block of iron at the same temperature have the same intensity of heat, but the larger block has twice the extension of total quantity of heat. Now motion itself fell under the same consideration, so that among the Oxford calculators speed came to be seen as the intensity of motion; that is, as an intensional quality with a latitude that could be intensified or remitted (diminished).

48 Clagett, *Science of Mechanics in the Middle Ages*, pp. 163–97.

49 Crosby, *Thomas Bradwardine*, IV, pp. 124–41.

50 James A. Weisheipl, "Ockham and some Mertonians," *Mediaeval Studies*, 30 (1968), 163–213; and Edith Dudley Sylla, "The Oxford Calculators," in Kretzmann, Kenny, and Pinborg, *Cambridge History of Later Medieval Philosophy*, pp. 540–63.

The intensity or speed of motion was distinguished from its extension in the same way that the intensity of a quality was distinguished from its extension. By analogy to hotness and other qualities, the motion of a body came to be measured both intensively as its speed at any instant and extensively as its total time and distance.[51]

Building on these ideas, William Heytesbury devoted Part VI of his comprehensive *Rules for Solving Sophisms* to problems concerning the three species of change: local motion, or change of place; augmentation, or change of size; and alteration, or change of quality. In the section on local motion, he distinguished uniform motion, which he defined as motion in which equal distances are traversed at equal speeds in equal times, from nonuniform or difform motion. Motion can be difform with respect to the subject, as for example the motion of a rotating body, since various points on the body move at various speeds; or with respect to time, as when a body is speeding up or slowing down; or with respect to both, as when a rotating body is altering its speed. Difform motion with respect to time can be uniformly difform (in modern terms, uniformly accelerated or decelerated), which Heytesbury defined as motion in which equal intensities of speeds are acquired or lost in equal times. Difformly difform (nonuniformly accelerated or decelerated) motions can vary in an infinite number of ways, which Heytesbury and his successors attempted in part to classify. In all cases, however, the speed of a motion, as well as its uniformity or difformity, is to be determined from the speed of the body's fastest-moving point. In the case of a motion difform with respect to time, speed at any instant is measured by the distance the fastest-moving point would have traveled if it had moved uniformly at that speed for some period of time.[52] Now Aristotle had insisted that there can be neither motion nor rest in an instant because motion and rest by definition occupy some finite time.[53] Heytesbury has thus defined, in defiance of Aristotle and common sense, instantaneous speed – an important step in the abstract treatment of motion that would culminate in the new science of Galileo and Newton.

The concept of instantaneous speed set the stage for the most significant contribution of the Oxford calculators to the kinematics of local motion – the mean-speed theorem. Its earliest known statement is found in Heytesbury's *Rules for Solving Sophisms*: a body uniformly accelerated or decelerated for a given time covers the same distance as it would if it were to travel for the same time uniformly with the speed of the middle instant of its motion,

[51] Clagett, *Science of Mechanics in the Middle Ages*, pp. 199–219; Edith Sylla, "Medieval Quantifications of Qualities: The "Merton School," *Archive for History of Exact Sciences*, 8 (1971/1972), 9–39; and Edith Sylla, "Medieval Concepts of the Latitude of Forms: The Oxford Calculators," *Archives d'histoire doctrinale et littéraire du moyen âge*, 40 (1973), 223–83.

[52] Curtis Wilson, *William Heytesbury: Medieval Logic and the Rise of Mathematical Physics* (Madison: University of Wisconsin Press, 1960), pp. 117–22; and Clagett, *Science of Mechanics in the Middle Ages*, pp. 235–7.

[53] Aristotle, *Physics*, VI.10.241a15–26.

which is defined as its mean speed. Thus a body accelerating uniformly for two hours will cover the same distance as would a body moving uniformly for two hours at the speed reached by the first body after one hour. From this Heytesbury drew several conclusions, including what is called the distance rule: that the distance covered in the second half of a uniformly accelerated motion from rest is three times the distance covered in the first half.

Heytesbury gave the mean-speed theorem without proof, but if he was the author of a related work on motion entitled the *Proofs of Conclusions*, he is responsible for a common proof repeated by a number of his successors, including Richard Swineshead and John Dumbleton. In this proof, a uniformly accelerated motion is divided at its middle instant into two halves, and each instant in the first half corresponds to an instant in the second half. Since the speed at each instant in the slower half is less than the mean speed by exactly as much as the speed at the corresponding instant in the faster half exceeds the mean speed, the same overall distance is covered as would be covered by a uniform motion at that mean speed.[54]

Like Bradwardine's theorem, the methods and results of the other Oxford calculators spread to the continent over the next generation, appearing most notably at the University of Paris in the works of Albert of Saxony, Nicole Oresme, and Marsilius of Inghen. The most significant addition to the Oxford subtleties was Nicole Oresme's graphical representation of qualities and speeds, including a graphical proof of the mean-speed theorem. Although the method of representing qualities graphically perhaps occurred earliest in Giovanni di Casali's *On the Speed of Alteration*, dated 1346 in one manuscript, Oresme, at about the same time or somewhat later, developed a graphical method of considerable power and flexibility. Whereas the Oxford calculators had represented the intensities of qualities and speeds with lines of various lengths, Oresme, in his *On the Configuration of Qualities*, represented them with figures of two or more dimensions. A horizontal line – called the subject line – represented the subject or body, from each point of which was raised a vertical line representing the intensity of some quality at that point (see Figures 17.1 and 17.2). The line joining the tops of the intensity lines – called the line of summit or line of intensity – defined what Oresme called the configuration of the quality. In the case of a quality uniform over the length of the subject line, the configuration is rectangular (Figure 17.1); in the case of a quality uniformly increasing from zero at one end of the subject line to some finite intensity at the other, it is triangular (see Figure 17.2), and so on. Oresme also used the subject line to represent the time of a motion and the intensity lines to represent its speed at each instant. A uniform motion is thus represented by a rectangle, and a uniformly accelerated motion by a right triangle. Oresme called the area of such a figure the total velocity of the

[54] Wilson, *William Heytesbury*, pp. 122–3; and Clagett, *Science of Mechanics in the Middle Ages*, pp. 255–329.

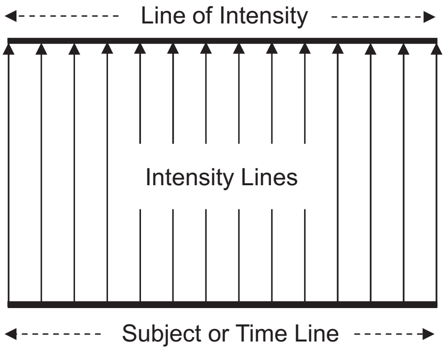

Figure 17.1. The configuration of a uniform quality or motion.

motion since it is composed of all the infinite number of velocity lines within it. He also recognized that the total velocity was dimensionally equivalent to the distance traversed by the motion.

From this the mean-speed theorem and the distance rule immediately follow, for it is evident that the area of a right triangle, representing the total intensity of a quality distributed uniformly difformly from zero to some intensity, is equal to that of the rectangle on the same base with half the altitude, representing a uniform quality at the mean degree. And since the area of the triangle representing the slower half of the motion is one-fourth the area of the whole triangle representing the whole motion, the distance rule – that the distance covered in the faster half of the motion is three times that covered in the slower – is also confirmed. In his *Questions on Euclid's Elements*, Oresme in effect generalized the distance rule, showing that for

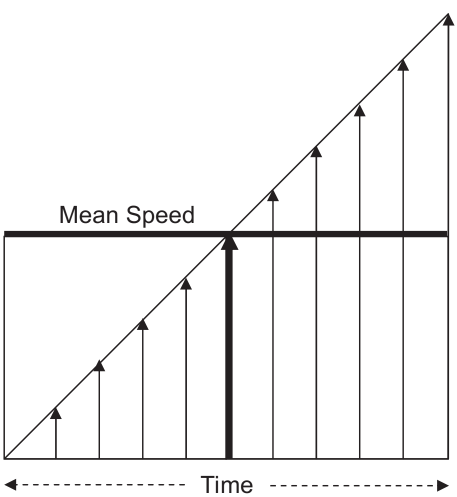

Figure 17.2. The configuration of a uniformly difform motion and the mean speed.

qualities in a uniformly difform distribution from zero to some intensity, the total intensity in each successive equal part of the subject follows the sequence of the odd numbers (1, 3, 5, etc.). If Oresme had demonstrated this specifically for equal times in a uniformly accelerated motion, he would have expressed the odd-numbers rule – that distances covered in equal times vary as the odd numbers – which would later prove to be a central proposition in Galileo's new science of motion.[55]

The main purpose of Oresme's configuration of qualities was not to develop a kinematics, however, but rather to explain surprising and unexpected physical effects by appealing not simply to the intensity of a quality in a subject but also to its configuration. He explicitly recognized that these configurations were imaginary or hypothetical, and in devising them he did not rely on any actual measurements. His extension of configuration theory to motion seems to have been a legacy from the Oxford calculators, where motion was one of many topics in natural philosophy that provided material for the paradoxes and logical puzzles that made up the *sophismata* of the arts curriculum. And, in general, natural philosophers from the fourteenth century on were more concerned with classifying types of uniform and difform motions and compiling solutions to various hypothetical situations than with discovering the way things occur in nature.

CELESTIAL MOVERS

Within the Aristotelian cosmos, the sublunary realm – the spheres of the four elements, located below the Moon – was the realm of motion and change, augmentation and diminution, growth and decay. Above the Moon, by contrast, was the celestial realm, occupied by bodies as far surpassing elemental bodies in perfection and splendor as they are distant from them. In *De caelo*, Aristotle suggested that the celestial spheres should be thought of as living bodies composed of a celestial material called ether or, as it was later known, the fifth essence, or quintessence. Unlike the four elements, which move naturally with rectilinear motions up or down, the ether moves naturally with circular motion, as the daily rotation of the heavens reveals. And since circular motion has no contrary, Aristotle inferred that the celestial spheres were ungenerated and incorruptible, fully actualized and without any potential for change beyond their natural and eternal circular motions.[56]

The existence of motion in the cosmos as a whole implied for Aristotle the existence of realities that lie beyond the material. From the premise that

[55] Clagett, *Science of Mechanics in the Middle Ages*, pp. 331–418; and Marshall Clagett, *Nicole Oresme and the Medieval Geometry of Qualities and Motions: A Treatise on the Uniformity and Difformity of Intensities known as Tractatus de configurationibus qualitatum et motuum* (Madison: University of Wisconsin Press, 1968).

[56] Aristotle, *De caelo*, I.1–2.269a18–b17; II.2.285a29–30; and II.12.292a19–22.

everything that is moved is moved by another, he proved in Book VIII of *Physics* that there must be a first mover and a first motion – not first in time, for Aristotle held that the world and its motion were eternal, but first in priority. This first motion, he argued, must be the uninterrupted rotation of a single body, the first or prime mobile (*primum mobile*), moved by a single mover, the first or prime mover (*primum movens*), acting on it with absolute uniformity. The prime mobile must be a physically real celestial sphere, above the sphere of the fixed stars, which receives its own perfectly uniform and regular motion from the first or prime mover and somehow communicates that motion to the sphere of the fixed stars below and from there to the spheres of the planets, Sun, and Moon. The prime mover must itself be immaterial and unmoved, moving the prime mobile as its final cause (its goal or purpose).[57] In his *Metaphysics*, Aristotle suggested that the lower planetary spheres, each of which has its own proper motion, are also moved by unmoved movers, each acting as an external final cause.[58] For Aristotle, then, the existence and persistence of motion in the world implied the existence of an eternal and immaterial unmoved mover as its first cause, an argument that Aquinas adopted as the first of his famous five proofs for the existence of God.[59]

How the prime mover moves the prime mobile, how that motion is passed on to the spheres below, and how each of the lower spheres has, in addition, its own proper motion gave rise to much controversy among medieval commentators. That the celestial spheres were living bodies possessing animating souls was greeted with skepticism, especially after the opinion was condemned in 1277. If, on the other hand, they were inanimate, how could they be moved by nature, since they were already located in their natural places? Furthermore, since their circular motion was infinite (that is, without any definite end or goal), it could not be natural since it would be incapable of fulfillment and would thus be in vain; and nature, according to Aristotle, does nothing in vain. Faced with these perplexities, many took up Aristotle's suggestion of an external, unmoved mover for each celestial sphere, though there was no agreement on whether these unmoved movers – called celestial intelligences and sometimes identified as angels – moved the heavenly bodies directly or through the mediation of purely intellective (rather than animating) souls.[60] One alternative to celestial intelligences was to suggest, as Buridan did, that once the celestial spheres had been created and set in

[57] Aristotle, *Physics*, VIII.1–10.250b11–267b26; and Aristotle, *Metaphysics*, XII.7.1072a19–1073a13.

[58] Aristotle, *Metaphysics*, XII.8.1073a14–1074b14.

[59] Thomas Aquinas, *Summa theologiae*, prima pars, qu. 2, art. 3; and Thomas Aquinas, *Summa contra gentiles*, I.13.

[60] James A. Weisheipl, "The Celestial Movers in Medieval Physics," *The Thomist*, 24 (1961), 286–326, reprinted in Weisheipl, *Nature and Motion in the Middle Ages*, pp. 143–75; and Edward Grant, *Planets, Stars, and Orbs: The Medieval Cosmos, 1200–1687* (Cambridge: Cambridge University Press, 1994, repr. 1996), pp. 469–87, 514–48.

motion by God, in the absence of any resistance their impetus would continue to move them perpetually. This alternative was taken up by very few, however. Only Albert of Saxony seems to have adopted impetus as the mover of the heavens, most others preferring some version of celestial intelligences. Nicole Oresme, for one, objected that the celestial material was incapable of receiving impetus, perhaps because it was neither heavy nor light.[61]

The many remarkable developments that took place within medieval treatments of motion arose mainly out of the reading and interpretation of Aristotle's works, although they often took very un-Aristotelian turns and sometimes challenged fundamental tenets of Aristotelian natural philosophy. At the core of that natural philosophy was the premise that motion and change are the characteristic expressions of the natures of physical bodies, where nature – the internal source of motion and change within natural things – always acts for a purpose, toward a definite end or goal that fulfills or perfects the body moved. Medieval natural philosophers examined the nature and reality of motion in the light of both Aristotelian philosophy and newer philosophical positions, such as Ockham's nominalism, and they employed hypothetical cases, such as motion in a void, as a way of scrutinizing motion and discovering its essential characteristics. Dissatisfied with Aristotle's explanation of projectile motion, natural philosophers devised impetus theory and then applied it to naturally falling bodies, a move that tended to blur Aristotle's fundamental distinction between natural and violent motions. Similarly, the suggestion that the celestial spheres could be moved by impetus implies a blurring of the distinction between the celestial and elemental realms.

Aristotle's application of mathematics to motion had been rudimentary at best. Medieval logicians and natural philosophers brought to bear on motion new mathematical methods and techniques of great sophistication, which in turn yielded a number of conceptual insights. The first of these was Bradwardine's rule, with its application of ratio theory to motion, implying a recognition of the idea of a functional dependence between powers, resistances, and speeds. With Bradwardine and the Oxford calculators in general, motion came to be considered in abstraction from physical bodies and their natural tendencies, and speed came to be regarded as a magnitude that could be measured both intensively and extensively. Once they had grasped the paradoxical notion of instantaneous speed, the Oxford calculators and their followers could compare the speeds of various motions, both real and hypothetical, in a variety of sophisticated ways, most notably by applying the mean-speed theorem. With Oresme's powerful method of graphing qualities and motions, results such as the mean-speed theorem and the distance rule became immediately obvious. Motion and change also provided material for

[61] Clagett, *Science of Mechanics in the Middle Ages*, pp. 536, 561, 570; and Grant, *Planets, Stars, and Orbs*, pp. 548–52.

purely logical inquiries that used new quasi-mathematical methods of analysis to treat problems concerning infinity, beginnings and endings, maxima and minima, and first and last instants, which, among other things, yielded new mathematical insights into infinity and continuity.[62]

The study of motion in the Middle Ages, then, was not a slavish and sterile commentary on the words of Aristotle, but neither was it a failed attempt at the experimental science of motion that Galileo, Descartes, and Newton would establish in the seventeenth century. Rather, from a critical examination of the best sources available at the time, medieval logicians, philosophers, and theologians undertook to explain motion and change and the many puzzles surrounding them, developing in the process a series of new insights and analytic techniques that yielded a number of notable results. Part of the measure of their success – but only part – is that some of these insights and results had to be rediscovered later by Galileo and others in the course of the Scientific Revolution.

[62] See John E. Murdoch, "Infinity and Continuity," in Kretzmann, Kenny, and Pinborg, *Cambridge History of Later Medieval Philosophy*, pp. 564–91 at pp. 585–90.