Who Shares Risk with Whom and How? Endogenous Matching and Selection of Risk Sharing Equilibria

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Abstract

I examine a model in which heterogeneous agents first form risk-sharing pairs, and then repeatedly share income. I explore the interplay between matching patterns and the (selection of) equilibria of the risk-sharing game. Agents are heterogeneous in income autocorrelations, i.e. how their current incomes correlate with their past incomes. Agents with high autocorrelations are hard to share risk with: if she needs money now, then she is likely to need money again in the next period, and thus is unlikely to repay it soon, diminishing her partners incentives to lend to her; similarly, in periods in which she has a high income, she is unlikely to need to borrow for a long time, diminishing her incentives to lend to her partner. With endogenous matching, all agents prefer to match with partners who have low income autocorrelation. The resulting equilibrium features substantial inequality and low total welfare compared to what would happen if the agents could be forced into a specific matching but shared risks voluntarily. Either agents match suboptimally, so agents match within-type, which leads to inequalities across pairs and hurts the population on average, or they match optimally, but in order to sustain cross-type matches agents end up with unequal risk sharing arrangements, which results in lower risk sharing levels. In an extension I show that common income shocks can change matching patterns, which, paradoxically, may improve overall risk sharing and reduce inequality. My analysis also applies to other sorts of heterogeneities, such as heterogeneous opportunities to rematch or migrate.

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1 Introduction

Informal risk sharing is important, but imperfect.[1] Standard economic theory suggests that people should share risk with others that are different from themselves, so as to avoid correlations in their income and maximize the risk-sharing potentials. However, it is well-documented that people tend to share risk with similar others.[2] Why is this the case?[3] This paper provides one theoretical force to explain this pattern, based on the observation that similar agents have common features[4] that have substantial effects on agents' incentives in risk-sharing and consequently on the matching patterns.

To do so, I study a model where heterogeneous agents first form risk-sharing pairs, then share risk infinitely repeatedly under limited commitment.[5] One novelty of the paper is to endogenize the matching process before informal risk sharing, which allows me to explore the interplay between matching patterns and the (selection of) equilibria of the risk sharing game. Agents are heterogeneous in income autocorrelations, i.e. how their current incomes are correlated with their past incomes. The income autocorrelation is often driven by occupations: accoding to the European Community Household Panel (ECHP, 1994-2001), autocorrelation in annual income has a median of 0.05 for "agricultural, fishery and related labourers", compared to a median of 0.55 for "laboureres in mining, construction, manufacturing and transport".[6]

To illustrate this idea, consider a village in which each agent has high income in half of the periods and low income in the other half. There are two occupations: fishermen (f) and

[1]Pioneered by Townsend (1994), many papers reject the null hypothesis of full risk-sharing. See, for example, Deaton (1992) for Cote d'Ivoire, Ghana and Thailand, Udry (1994) for northern Nigeria, Grimard (1997) for the Cote d'Ivoire, Dubois (2000) for Pakistan, Fafchamps & Lund (2003) for the Philippines, etc.

[2]See, for example, Fafchamps & Gubert (2007) for risk-sharing within occupations, Ghatak (2000) that documented the self-selection of individuals/households into different risk pools, and Mazzocco & Saini (2012) for risk-sharing within caste. For general evidences of "homophily", i.e. the tendency of people to associate more with others that are similar to them, see McPherson (2001) and Jackson (2008) for extensive reviews.

[3]Besides the explanation provided in this paper, there can be various alternative explanations. One is based on information asymmetry. Another is culture-based, for instance, see Jackson & Xing (2014) for an experimental study that justifies homophily based on the ease of coordination among people that have the similar cultural background.

[4]In most parts of the paper I focus on the case in which agents are heterogeneous in autocorrelations in income; in extensions I show that the model applies to a class of alternative heterogeneities based on time preferences, income variances, wealth levels, and options to migrate/rematch, etc.

[5]Several papers provide empirical evidences for the importance of limited commitment in risk sharing. See, e.g. Foster & Rosenzweig (2001), Fafchamps & Lund (2003), and De Weerdt & Dercon (2006).

[6]An autocorrelation of 0.05 corresponds to a probability of 52.5% for having the same income in two consecutive periods and an autocorrelation of 0.55 corresponds to a probability of 77.5%, both in context of the following example assuming that income follows a Markov process.

coal miners (m). A fisherman looks for fish in every period and his/her income is roughly independent over time. Whereas for a miner, getting a high income in one period implies a larger chance of getting a high income again in the next period: once she finds a vein to work in she gets high income, which remains for a while until the vein runs out; then the miner needs to find a new vein, and the fact that those veins are hard to find means the miner may receive low income for a long period of time. All agents are risk averse and look for a partner to share risk. In particular, risk-sharing is in a manner that when one partner has a high income and the other has a low income, the former can help out the latter by making a transfer. Who shares risk with whom - will a fisherman match with another fisherman or with a miner? How do they share risk? What are the consequences of matching in terms of inequality and total welfare?

Positive income autocorrelations do not change the opportunities for risk-sharing, yet reduce agents' incentives. The frequency of dates at which one partner can help out the other is a half,[7] *regardless of the agents' income autocorrelations.*[8] However, one's continuation value from the relationship, *conditional on* helping out the other (i.e. one has a high income and the other has a low income), substantially depends on agents' autocorrelation types. In particular, when an agent's income is highly autocorrelated, if she needs money now, then she is likely to need money again in the next period, and thus is unlikely to repay it soon. This, together with impatience, implies a lower continuation value for the agent's partner (the lender) and thus her partner has less incentives to lend. Similarly, an agent's high autocorrelation not only hurts her partner's incentives, but also her own incentives to lend: in periods when she has a high income, she expects a string of high income in the near future and hence is unlikely to need to borrow for a long time, which lowers her incentives to lend. In sum, partners who have high autocorrelations are difficult to share risk with.

The above leads to a tendency of positive assortative matching. Everyone prefers to share risk with a partner who faces a less autocorrelated income. Fishermen prefer to share risk with other fishermen. Miners, although willing to share risk with fishermen, have to share risk among themselves.

This matching pattern leads to substantial inequality in terms of the amount of risk faced by the agents: fishermen share risk with other fishermen and they share risk well; whereas for a pair of coal miners, they both have high income autocorrelation thus share risk badly. Such an inequality inequality in risk exposure appears despite the fact that agents of all types have exactly the same average income and income variance.

[7] I assume that income is independent across agents so as to get a clean benchmark. Then in Section 4 I consider interdependence across agents.

[8] This implies that set of the feasible payoffs remains the same regardless of the agents' income autocorrelations. In other words, if agents were able to commit to risk sharing arrangements, then income autocorrelations would have no impact on agents' ex-ante payoffs.

Both the matching pattern and the selection of risk-sharing equilibrium can be sub-optimal. On one hand, compared to the case where all agents share risk across-type, when agents match positive assortatively, one type (fisherman) gets excessive amount of risk sharing and the other type (miner) gets too little risk sharing. This, together with the decreasing marginal benefit of risk sharing, leads to a reduction in total welfare.[9] On the other hand, if agents match optimally, then in order to sustain that matching (across-type) agents have to end up with very unequal sharing arrangements within a partnership, which in turn lowers risk sharing levels.

A more concrete overview. My model proceeds in two phases. First, in the matching phase, agents match in pairs and the matching is once and for all.[10] Then in the risk-sharing phase, each pair of agent share risks infinitely repeatedly under limited commitment. Limited commitment means that agents cannot commit to future risk-sharing arrangements, so that making a transfer in any period must be incentivized by continuation benefits in the future. Under limited commitment, a pair of agents need to adopt a risk-sharing arrangement that is subgame perfect,[11] and such a risk-sharing arrangement is called a risk-sharing equilibrium.

An equilibrium in this two-phase game is a matching function and a collection of risk-sharing arrangements, one for each matched pair, such that (1) the risk-sharing arrangement for each pair is subgame perfect, and (2) no two agents can form a blocking pair, i.e. match with each other and find *some* subgame perfect risk-sharing arrangement that Pareto dominates the payoff they currently receive. Since any risk-sharing equilibrium can be adopted to form a blocking pair according to the equilibrium notation, I need to characterize the whole equilibrium risk-sharing payoff frontier, which depends on the corresponding pair of agents' (autocorrelation) types.

There is a risk sharing equilibrium in which a pair of agents adopt the largest amounts of transfers that are self-enforcing whenever transfers are needed.[12] This equilibrium is called the maximum transfer equilibrium (MTE). It is this equilibrium, among all risk sharing equilibria, that achieves the largest amount of transfers and maximizes the sum of agents'

[9] Here I take a utilitarian perspective and consider the total welfare as the sum of all agents' (ex-ante) equilibrium payoffs.

[10] The matching phase is modeled as a "roommate problem", i.e. any two agents can potentially form a pair. (See, e.g. Gale & Shapley (1962) and Chiappori et. al. (2014).) This is different from the "two-sided matching" models in which the population has two subgroups (e.g. men and women) and matching can only occur across the groups. The roommate problems better capture the nature of matching in risk sharing, and in addition provides additional structure in predicting the selection of risk sharing equilibrium.

[11] This paper will mostly focus on a simple set of equilibria, the "semi-Markovian" equilibria, for which the subgame perfection is equivalent to "self-enforcing" that is widely adopted in the risk-sharing literature (see, e.g. Ligon, Thomas & Worrall (2002)).

[12] i.e. when the two agents have different levels of income.

ex-ante payoffs. In addition, *neither* agent can use a larger transfer in any other equilibrium. This is due to a complementarity in incentive structures: one has no incentive to transfer more while expecting a less amount from his/her partner. Finally, the MTE is also the equilibrium that minimizes inequality between the pair of agents among all equilibria that are Pareto undominated.

Despite the nice properties of the MTEs, when endogenous matching is accounted for, every pair of agents with different types is forced to select some equilibrium that leads to uneven payoffs between the agents, favoring the less autocorrelated. This is because that agents of different types face different outside options, each of which is defined as the payoff from a within-type pair (with the corresponding MTE played). Such an uneven arrangement leads to reductions in both agents' transfers, a distortion in the sum of agents' payoffs, and an increase in inequality.

The above issue makes across-type pairs hard to be sustained in equilibrium, and reinforces the matching patterns in which agents share risk with others of the same type. I provide a simple sufficient condition for such matching patterns based on the relative positions of risk-sharing payoff frontiers and the effectiveness of transfer risk sharing payoffs between a pair of agents. This analysis sheds light on the origins of homophily; a ubiquitous pattern of people to interact more with others who are similar to themselves.

Extensions. The first extension concerns what happens if agents' income flows are correlated. In particular, agents with the same type are affected by type-specific common shocks, so that they have positively correlated income. Incomes for different types of agents remain to be independent. [13] Common shocks reduce the risk sharing potentials for every within-type pair. Therefore, when the frequency of common shocks increases, risk sharing performances become worse for a within-type pair, while it remains unchanged for an across-type pair. In the presence of less-frequent common shocks, the pattern of sharing risk with similar others remains in equilibrium and is more inefficient. Surprisingly, when the common shocks are frequent enough, an increase in their frequency may improve total welfare. These positive welfare effects of common shocks come about through a change in the endogenous matching pattern and a reduction in the asymmetry (i.e. the departure of selected equilibrium from the MTE) in risk-sharing among heterogeneous types.

The second extension is to explore alternative heterogeneities. In particular, agents are heterogeneous in 1) their wealth levels, 2) their income variances, or 3) their patience. This list covers both the differences in environments that different agents may face (the first

[13]This captures the idea that agents similar to each other (e.g. in occupations) tend to have larger interpersonal correlations between their incomes. For instance, a storm keeps all fishermen at home and all of them get low income during that period.

two) and the difference in agents' preferences (the third). In all the above, the equilibrium payoff frontier in risk sharing moves monotonically in types and therefore everyone has the same preference for their partner's types: everyone likes a wealthier partner, everyone likes a partner with more variable income, and that everyone likes a more patient partner. Results about the interplay between matching and risk sharing are parallel to those presented in the case where agents are heterogeneous in autocorrelations. The analysis with alternative heterogeneities shed lights on several empirical findings, including that the rich receive more from (rather than give to) the poor (Misrut (2008)), and the self-selection of individuals into different risk pools (Ghatak (2000)).

The third extension allows agents to opt out of their current relationship and match with other people. Migration is one example of the rematch process: agents migrate and form new risk-sharing partnerships.[14] A more effective rematch process, measured by a shorter waiting before being matched again, can actually make everyone worse off. This is due to a negative effect of rematching on risk-sharing: the opportunity of rematching results in a smaller cost in breaking the current relationship, and thus reduces agents' incentives to make a transfer. In addition, when agents are heterogeneous in their opportunities of rematch, everyone prefers to share risk with a partner with worse rematch options and consequently there is a tendency of sorting with respect to the opportunity of rematch.

Related literature. This paper adds to several branches of literature, besides the empirical literature already cited in this introduction.

First, the paper is related to the literature on the imperfect risk sharing with fixed risk-sharing pairs.[15] Those papers assume agents are born into their risk sharing groups and cannot choose with whom to share risk. In contrast, my paper explicitly models the matching process among hetergeneous agents. In addition I show that endogenous matching affects the selection of the risk sharing arrangements, which in every across-type pair leads to further distortions in both agents' transfers.

Another related branch of the literature is refers to matching with heterogeneous agents in the absence of subsequent strategic interactions.[16] Related to risk sharing, existing papers mainly focus on matching among agents that are heterogeneous in risk preferences.[17]

[14] For migration and consumption smoothing, see, e.g. Rosenzweig & Stark (1989), Morten (2013), and citations therein.

[15] These include hidden income (Cole & Kocherlakota (2001)), moral hazard (Phelan & Townsend (1991)), and limited commitment (Coate & Ravallion (1993), Kocherlakota (1996), Fafchamps (1999), and Ligon, Thomas & Worrall (2002)). The risk sharing part of our paper builds upon the limited commitment models, esp. Ligon, Thomas & Worrall (2002), who also allows for auto-correlations.

[16] This literature is pioneered by initiated by Gale and Shapley (1962) & Becker (1972), and later advances include Roth & Sotomayor (1989), Avery & Levin (2010), Chade et al. (2014), etc.

[17] See, e.g. Schulhofer-Wohl (2006), Legros & Newman (2007), Chiappori & Reny (2015) and Wang (2015)

This paper differs from the above in three main aspects. First, all the above assume full commitments in the post-matching stage, whereas I relax this assumption and explicitly analyze the incentive issues that are salient in risk-sharing. Second, this paper models the matching phase as a "roommate problem", i.e. any two agents can potentially form a pair.[18] Such a framework better captures the nature of matching in risk sharing, and also provides additional structure in predicting the selection of risk sharing equilibrium. Third, while the literature focuses on heterogeneity in risk preferences which often leads to negative sorting, this paper explore novel sources of heterogeneities (autocorrelations in the main model, then income levels, income variances, time preferences and options of rematch/migration as extensions). With any of these heterogeneities, all agents have similar preferences over the partner's types, and as a result there is a tendency for positive assortative matching. Thus this paper provides an new perspective on the origin of homophily.[19]

There is also a branch of literature on endogenous partnership formation with homogeneous agents.[20] These papers provide important insights on the sizes and shapes of partnerships, yet cannot answer the question of "who interacts with whom". This question is better explored in our framework with heterogeneous agents. As for how partnership formation affects the play of the subsequent game, our paper closely relates to Jackson & Watts (2010), but identifies a very different key determinant for the selection of equilibrium: it is the relative advantage in risk sharing between different types in this paper, in contrast to the relative population size in Jackson & Watts (2010).[21]

The rest of the paper proceeds as follows: Section 2 presents the model and equilibrium notions. Section 3 characterize features in risk-sharing equilibria, their selection when

for theoretical discussions, and Ackerberg & Botticini (2002) for empirical evidences.

[18]See, e.g. Gale & Shapley (1962), Gusfield & Irving (1989), Klaus & Klijn (2010), Irving (1985), Tan (1991) & Chung (2000) for roommate problems with non-transferable utilities; and Karlander & Eriksson (2001), Klaus and Nichifor (2010), Talman & Yang (2011), Chiappori et. al. (2014) for the cases with perfectly transferable utilities. This paper can be viewed as a roommate problem with utilities partially transferable along the frontiers, which are shaped by the subsequent risk-sharing phase. The risk sharing game provides regularities on how the payoff frontiers depend on the combination of types, and thus adds tractability to the roommate problem.

[19]See, e.g. McPherson et. al. (2001), Jackson (2008), Currarini et. al. (2009) and the extensive literature reviews therein.

[20]For instance, Genicot & Ray (2003) show that stable groups are of bounded sizes when allowing for coalitional deviations. Bloch et.al. (2008) studies stable risk sharing networks. Jackson & Watts (2010) establish a framework in which agents form groups to play games. Agents are divided into several "roles" and each group requires one agent from each role. In their paper, agents in a role are identical.

[21]For more papers studying the implications of endogenous partnership formations for behavior, see, e.g. Kelso & Crawford (1982), Rubinstein & Wolinsky (1985), and Hatfield & Milgrom (2005) for market and bargaining settings, Watson (1999) for mutual investment, Goyal & Vega-Redondo (2005) for coordinations, and Ghosh & Ray (1996) for cooperations in prisoners dilemma games.

matching is endogenized, implications for total welfare, and conditions for positive assortative matching. Section 4 introduces common shocks for agents who have the same type and discusses their welfare effects. Section 5 explores three alternative heterogeneities. Section 6 allows for rematch. Section 7 concludes.

2 Model: Matching and Repeated Risk Sharing

In this section I present the model. A population of agents, denoted by N, act in a two-phase game. In phase one (the matching phase) agents form bilateral risk-sharing pairs; then in phase two each pair of agents share risks with each other infinitely repeatedly under limited commitment. Before describing the two phases in more detail, I first describe the income patterns that agents face and the source of heterogenity.

At each date $t = 0, 1, \ldots$, each agent i receives an income (amount of consumption good, also referred to as "money") $s_i^t \in \{0, 1\}$, in which 0 represents a low income and 1 a high income. The consumption good is non-storable. I assume income shocks are independent across agents, until Section 4 in which common shocks are introduced.

An agent's income flow can be (auto-)correlated over time, and agents are heterogeneous in the time-patterns of income flows. In particular, an agent's income flow $(s_i^0, \ldots, s_i^t, \ldots)$ follows a Markov process with state space $\{0, 1\}$ and transition matrix $\mathbf{\Pi}_i \equiv \begin{bmatrix} \theta_i & 1-\theta_i \\ 1-\theta_i & \theta_i \end{bmatrix}$, in which $\theta_i \in [0.5, 1)$ is called the agent's *autocorrelation type*. An agent has i.i.d. income if $\theta = 0.5$, has positively correlated income if $\theta > 0.5$; and I assume no one has a negatively correlated income flow. Throughout the paper I assume the number of agents in each type is even.[22]

To focus on the effects of autocorrelation, I assume that agents are the same in other dimensions. In particular, all agents share the same expected income (per-period): $\Pr(s_i^0 = 0) = 0.5, \forall i$ so that the two levels of income are equally likely ex-ante and both have a frequency of 0.5. Agents also have the same preference. In particular, agent i has per-period von Neumann-Morgenstern utility $u(c_i^t)$ in which c_i^t is the consumption of agent i. Agents are risk averse: $u'(c) > 0, u''(c) < 0, \forall c$. Agents discount the future with common discount factor $\delta \in (0, 1)$, and are expected utility maximizers.

Agents form risk-sharing pairs at the beginning of $t = 0$ before the time-0 income is realized. A matching fuction is denoted by $m : N \to N$, with $m(i)$ representing the agent with whom agent i is matched. The partnerships are bilateral so that $j = m(i) \iff i = m(j)$. The match is once and for all, so agents are not able to rematch in the future. Agents commonly observe each other's autocorrelation types at the time of matching, and each agent

[22]The number of agents is even so that it is feasible to have all agents match within their types.

faces the rest of the whole population as potential partners.

Each pair of agents (i, j) s.t. $i = m(j)$ share risks with each other. The risk-sharing relationship is modeled as an infinitely repeated stochastic game, with the agents' (joint) income as exogenous states. By construction, the sequence of (joint) income states follow a Markov process with state space $S \equiv \{(1,1), (1,0), (0,1), (0,0)\}$ and transition matrix $\mathbf{\Pi}(\theta_i, \theta_j) = \mathbf{\Pi}_i \otimes \mathbf{\Pi}_j$, i.e. the Kronecker product of the two transition matrices for individual income states.

At each date $t = 0, 1, \ldots$, agents can make transfers to each other after the realization of their income shocks, which are commonly observable. Formally, an action $q_i^t \in [0, 1]$ is the amount of transfer made by agent i to his/her partner. A strategy of agent i assigns an action to each decision node, i.e. $\mathbf{q}_i : S \times \bigcup_t H^t \to \mathbb{R}_+$, in which S is the set of income states and H^t is the set of histories up to time t, with a representative element $h^t = \left(s_i^\tau, s_j^\tau, q_i^\tau, q_j^\tau\right)_{\tau=0,\ldots,t-1}$. An agent's consumption is his/her income minus the net transfer to the other. In particular, agent i's expected payoff from a strategy profile $(\mathbf{q}_i, \mathbf{q}_j)$ is

$$U_i(\mathbf{q}_i, \mathbf{q}_j) \equiv \sum_{t \geq 0} \sum_{s^t, h^{t-1}} \delta^t \operatorname{Pr}(s^t, h^{t-1}) \; u\left(s_i^t - \mathbf{q}_i(s^t, h^{t-1}) + \mathbf{q}_j(s^t, h^{t-1})\right).$$

There is no information asymmetry: each pair of agents commonly observe their transfers in the past, and past and current income states. Therefore limited commitment is the only friction that agents face in risk-sharing. In particular agents cannot commit to (or find a third party to enforce) further risk-sharing arrangements, so that at any point of the risk sharing phase one's incentives of transfering money to the other must be provided by the expectation in continuation gains from the relationship. In particular notion of risk-sharing equilibrium requires that the risk-sharing strategy profile is subgame perfect.

An example.

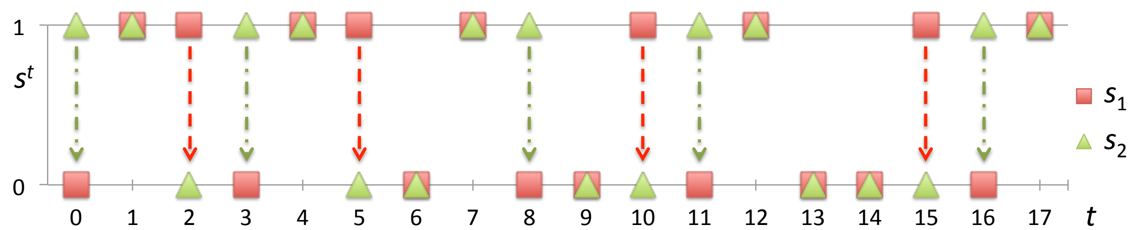
Before introducing the equilibrium notions, first consider an example highlighting why autocorrelation matters to risk-sharing. Suppose two agents have already been matched to share risk. The following example presents the time series of income realizations for agent 1 (red rectangles) and agent 2 (green triangles), with varying combinations of autocorrelation types. For instance, in period 0 of panel 1), agent 2 gets a high income and agent 1 gets a low one, and this is the situation in which 2 can help 1 out by delivering a transfer. I use the green dot-dashed arrows to represent the dates on which agent 2 can help agent 1, and the red dashed arrows to represent the dates on which agent 1 can help 2 (i.e. when 1 has a high income and 2 has a low income).

Among the total of 18 periods, the average number of periods in which one guy can help out the other (i.e. the arrow appears) is the same (9 in this example) regardless of the autocorrelations, due to the equal likelihoods of high and low income levels and independence across agents.

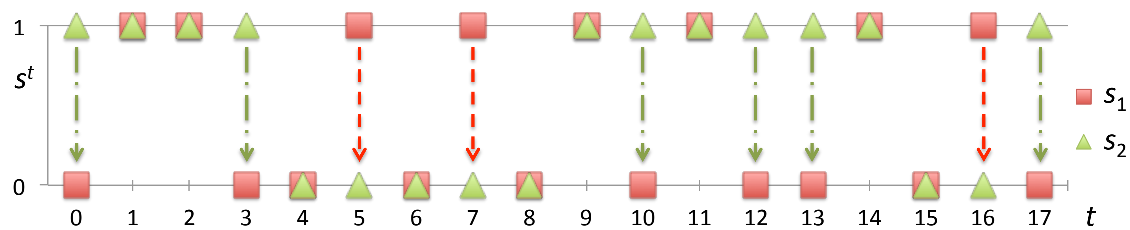
Nonetheless, incentives are particularly different across cases. First consider case 1) in which both agents' incomes are i.i.d. After agent 2 giving out his money in period 0, he gets some money back in period 2, which happens pretty soon. Whereas in case 2) when one agent has more autocorrelation and the other remains i.i.d., 2 gets a first transfer back later, in period 5. This is because agent 2's high income lasts for a while. Things are even worse in case 3) when both agents' incomes are positively autocorrelated: it takes a much longer wait for agent 2 to get a first transfer back (in period 8), since agent 2's high income and agent 1's low income both last for a while. Furthermore, before getting the first transfer back, agent 2 not only waits longer for the first transfer, but also expects to do more favors in case 2) and 3).

Therefore, in terms of the continuation value (of sustaining the relationship), it is much worse for agent 2 to deliver a favor in case 3) than in case 1); even though ex-ante they can share risk in the same number of periods.

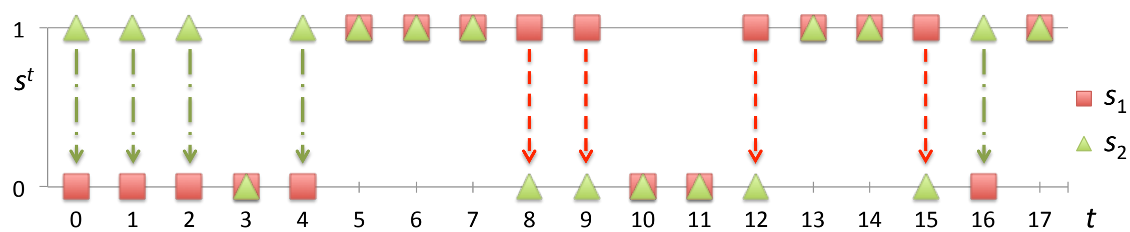
1) $\theta_1 = 0.5, \theta_2 = 0.5$: both have iid income flows.



2) $\theta_1 = 0.5, \theta_2 = 0.7$: the second agent has a postively autocorrelated income flow.



3) $\theta_1 = 0.7, \theta_2 = 0.7$: both agents have positively autocorrelated income flows.



The examples may seem specific but the insights are general: autocorrelations hurt incentives, and thus reduce the gains from risk sharing. I will more precisely present these insights after formally introducing the equilibrium notions.

2.1 Equilibrium Notions

In this part I first introduce the equilibrium notion in risk-sharing phase (taking matching patterns as given), then adopt it to define the equilibrium for the whole game including the matching and risk-sharing phases. Intuitively, an equilibrium consists of a matching function and a risk-sharing equilibrium for each pair, such that the matching is stable in the sense that any pair of agents (matched or unmatched) cannot find a risk-sharing equilibrium as a Pareto improvement for these two agents.

Semi-Markovian risk-sharing equilibria. For illustrative purpose I mainly focus on a class of strategies that is "history independent", i.e. each agent chooses the same amount of transfer at any point of time when a transfer is needed, with the only exception that when anyone deviates from the above, no further transfers will be made between the pair of agents (so that both agents are in autarky). One can view such strategies as depending only on the current state, including the income state and an endogenous state variable that keeps track of whether any deviation has occurred. In this sense, I call the class "semi-Markovian strategies". I also assume that transfers only occur when the pair of agents currently receive different levels of income, and in particular are made from agent who gets a high income to his/her partner who gets a low income.

Definition 1 (semi-Markovian strategy profile and equilibrium) *A strategy profile* $(\mathbf{q}_1, \mathbf{q}_2)$ *is* semi-Markovian *if there exists a pair of transfers* $(q_1, q_2) \in [0,1]^2$ *such that*

- $\mathbf{q}_i(s^0, h^{t-1}) \equiv q_i$ *if* $(s_i^t, s_j^t) = (1, 0)$ *and this condition is satisfied for history* h^{t-1}*,* $\forall \tau < t, i = 1, 2$*;*
- $\mathbf{q}_i(s^0, h^{t-1}) \equiv 0$ *otherwise.*

A semi-Markovian strategy profile, represented by its corresponding pair of transfers (q_1, q_2)*, is a* semi-Markovian equilibrium *if* $(\mathbf{q}_1, \mathbf{q}_2)$ *is subgame perfect.*

Matching game equilibria. Let $\Gamma(\theta_i, \theta_j)$ be the set of all (semi-Markovian) risk-sharing equilibria $(\mathbf{q}_i, \mathbf{q}_j)$, for a pair of agents with types (θ_i, θ_j). A matching game equilibrium is defined as follows:

Definition 2 (Matching game equilibrium) $\left(m, \{(q_i, q_j)\}_{m(i)=j, i<j}\right)$ *is a* matching game equilibrium *if*

1. *for any i and j matched ($m(i) = j$): $(q_i, q_j) \in \Gamma(\theta_i, \theta_j)$ is a risk-sharing equilibrium.*

2. *for any i and j, there exists no risk-sharing equilibrium (between i and j) that Pareto dominates what i and j currently receive.*[23]

A matching game equilibrium specifies a matching function, and a risk-sharing equilibrium for each matched pair. In addition, the matching should be "stable": on one hand, this requires that the any matched pair of agents the specified risk-sharing equilibrium is not Pareto dominated by any other risk-sharing equilibrium; on the other hand, this also requires that no pair of unmatched agents can find some risk-sharing equilibrium (when matched with each other) that Pareto dominates what they currently receive.

The notion of "stability" is quite strong in that it allows for any two agents to form a blocking pair with *any* risk-sharing equilibrium. As a result, in order to analyze the matching game equilibrium I need to fully characterize the equilibrium payoff frontier in the risk-sharing phase, for each pair of types.

3 Equilibrium Analysis

Recall that the model has two phases: agents first match in pairs and then risk-sharing takes place in each pair. This section characterizes equilibria using backward induction: first I characterize features of equilibrium payoff frontier in risk-sharing and illustrate how the frontier depends on the pair of types; then I analyze the equilibrium in the whole game. There is an interplay between matching and risk-sharing. First, endogenous matching determines the selection of risk-sharing equilibria for each pair. Second, equilibrium matching pattern is determined by the risk-sharing equilibrium frontiers.

3.1 Risk-sharing equilibria

I first characterize the risk-sharing equilibria for a given pair, say agents 1 and 2 with types θ_1, θ_2. Due to the lack of commitment, one's decision of giving out money needs to be incentivized by the expectation of future gains. I say a strategy profile is self-enforcing if none of the two agents has incentive to deviate under the punishment of staying in autarky, i.e. no further transfers are made from then on. This can be viewed as a "grim trigger" punishment in the risk-sharing environment.

[23] Formally, $\nexists(\widehat{q}_i, \widehat{q}_j) \in \Gamma(\theta_i, \theta_j)$ s.t. $U_i(\widehat{q}_i, \widehat{q}_j) \geq U_i(q_i, q_{m(i)})$ and $U_j(\widehat{q}_i, \widehat{q}_j) \geq U_j(q_j, q_{m(j)})$ and at least one inequality is strict.

Definition 3 (Self-enforcing transfers) *A risk-sharing strategy profile* $(\mathbf{q}_1, \mathbf{q}_2)$ *is* self-enforcing *if* $\forall i = 1, 2, \forall (s^t, h^{t-1})$:

$$U_i^t(\mathbf{q}_1, \mathbf{q}_2 \mid (s^t, h^{t-1})) \geq U_i^{Aut,t}(s^t, h^{t-1}) \equiv U_i^t(\mathbf{0}, \mathbf{0} \mid (s^t, h^{t-1})) \tag{1}$$

With semi-Markovian strategy profile (q_1, q_2), this can be re-written as ($\forall \{i, j\} = \{1, 2\}$)

$$u(1) - u(1 - q_i) \leq \sum_{\tau > 0} \delta^\tau \left[p^{give,\tau|give}(u(q_j) - u(0)) - p^{receive,\tau|give}(u(1) - u(1 - q_i)) \right], \tag{2}$$

in which ($\forall \tau = 1, 2, \ldots$)

- $p^{give,\tau|give} \equiv \Pr(s^{t+\tau} = (1, 0) \mid s^t = (1, 0))$, increasing in θ's;
- $p^{receive,\tau|give} \equiv \Pr(s^{t+\tau} = (0, 1) \mid s^t = (1, 0))$, decreasing in θ's.

In particular, the LHS in (3) is the cost of helping out the other in the current period t, and the RHS is the expected continuation value in sustaining the relationship (relative to the "outside option" of autarky). When either autocorrelation type increases, the likelihoods of the future gain $(u(q_j) - u(0))$ go down and the likelihoods of the future cost $(u(1) - u(1 - q_j))$ go up, hence either agent faces a tighter incentive constraint.

For any strategy profile, a necessary condition for subgame perfection is being self-enforcing, since each player can always guarantee himself the autarky payoff. For semi-Markovian strategy profiles, being self-enforcing is also sufficient condition for being an equilibrium.

Lemma **1** (q_1, q_2) *is a (semi-Markovian) risk-sharing equilibrium if and only if it is self-enforcing, i.e.* ($\forall \{i, j\} = \{1, 2\}$)

$$\frac{u(1) - u(1 - q_i)}{u(q_j) - u(0)} \leq \frac{\sum_\tau \delta^\tau p^{receive,\tau|give}}{\sum_\tau \delta^\tau p^{give,\tau|give} + 1} \equiv H(\theta_1, \theta_2), \tag{3}$$

in which $H(\theta_1, \theta_2)$ *decreases in both arguments.*

Autocorrelation hurts incentives. When either agent's income becomes more positively autocorrelated, what happens today better predicts the future. Hence an agent who is giving out money today expects a larger chance ($p^{give,\tau|give}$) to be giving out money again and a lower chance ($p^{receive,\tau|give}$) to be receiving money at any point of time in the future. Therefore, due to the expectation of a lower gain in the future, an agent has less incentive to give out money.

In above I have characterized the set of risk-sharing equilibrium transfers. What about equilibrium payoffs?

Notice that all pairs of agents face the same (ex-ante) chance of sharing risk, regardless of their autocorrelation types. In particular, the ex-ante frequencies for each agent to give out money and to receive money are both a quarter. Thus an agent's ex-ante payoff from a strategy profile (q_1, q_2) is:

$$U_i(q_1, q_2) = \frac{1}{4(1-\delta)} \left[u(q_j) + u(1-q_i) + u(1) + u(0) \right], \forall i = 1, 2. \tag{4}$$

This, together with Lemma 1, imply the following basic properties for the set of risk sharing equilibria and equilibrium payoffs.

LEMMA **2** *The set of risk-sharing equilibria* $\Gamma(\theta_i, \theta_j)$ *and the set of equilibrium payoffs* $\mathbb{U}(\theta_i, \theta_j)$ *are both compact and convex, and have the minimum* $(0, 0)$ *and* (U^{Aut}, U^{Aut})*, respectively,* $\forall(\theta_i, \theta_j)$*.*

In every pair, there is a *maximum transfer (risk-sharing) equilibrium* (MTE) $(q^*, q^*)(\theta_i, \theta_j)$:

$$q^* \equiv \text{argmax}_q U_i(q, q) \text{ s.t. } \frac{u(1) - u(1-q)}{u(q) - u(0)} \leq H(\theta_i, \theta_j)$$

The resulting (ex-ante) payoff

$$U^*(\theta_i, \theta_j) \equiv U(q^*(\theta_i, \theta_j), q^*(\theta_i, \theta_j))$$

is called the *maximum transfer equilibrium payoff* (MTE payoff). The symmetry in the set of equilibria also implies the symmetric structure in the MTE and its payoff.

LEMMA **3 (Maximum transfer equilibrium, MTE)** *In the risk sharing phase with every pair of agents* (i, j)*, the maximum transfer equilibrium* $(q^*, q^*)(\theta_i, \theta_j)$*, compared to any other equilibrium* $\forall(q_i, q_j) \in \Gamma(\theta_i, \theta_j)$*, achieves*

1. *a lower inequality:* $|U_i(q^*, q^*) - U_j(q^*, q^*)| \leq |U_i(q_i, q_j) - U_j(q_i, q_j)|$*;*
2. *a strictly higher sum of ex-ante payoffs:* $U_i(q^*, q^*) + U_j(q^*, q^*) > U_i(q_i, q_j) + U_j(q_i, q_j)$*;*
3. *strictly higher levels of transfers:* $q^*(\theta_i, \theta_j) > \max\{q_i, q_j\}$*.*

For the limited commitment to have non-trivial incentive effects, I assume that the agents cannot achieve the first-best transfer $q^{FB} = 0.5$ for any pair of types. Under this assumption, $q^*(\theta_i, \theta_j)$ is the maximum transfer used by both agents in any risk-sharing equilibrium. Moreover, it is also the maximum amount of transfer that can be used by *either* agent in every equilibrium. This is due to a complementarity in incentive structures: it is self-enforcing for one agent to transfer more when he expects a larger amount of transfer from his partner.

How do autocorrelations affect payoffs in risk sharing? An important lesson from equation (4) is that the autocorrelations have no direct effect to ex-ante payoff. They only affect agents' payoff indirectly through the incentives. Therefore, we have the following comparative statics of the set of equilibria (and their payoffs).

PROPOSITION **1 (Risk sharing equilibrium: comparative statics)** *The set of risk-sharing equilibria $\Gamma(\theta_i, \theta_j)$ and the set of equilibrium payoffs $\mathbb{U}(\theta_i, \theta_j)$ both shrink as either autocorrelation type increases:*

$$(\theta_i, \theta_j) \geq (\theta_i', \theta_j') \implies \Gamma(\theta_i, \theta_j) \subseteq \Gamma(\theta_i', \theta_j'),\ \mathbb{U}(\theta_i, \theta_j) \subseteq \mathbb{U}(\theta_i', \theta_j')$$

As for the MTEs: both the maximum transfer $q^*(\theta_i, \theta_j)$ and the MTE payoff $U^*(\theta_i, \theta_j)$ decrease in both arguments.

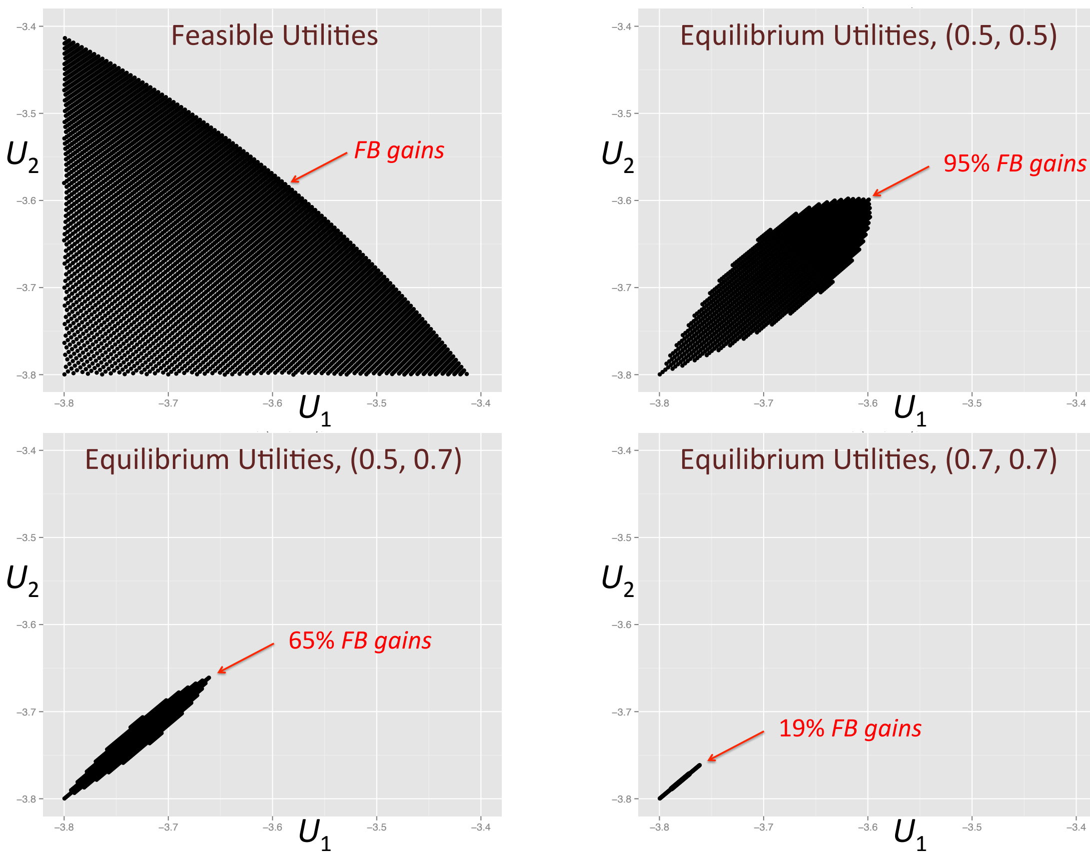
An increase in one's autocorrelation hurts not only this agent's own incentives, but also the others. This is because for one's switches role from giving out money to receiving money, both agents' income states need be flipped. The effects are symmetric, resulting a symmetry in the set of equilibria with respect to the 45 degree line.

So far, I have illustrated several properties of risk-sharing equilibria. First, in every pair, including one with different types of agents, there is a maximum transfer equilibrium which has the largest amount of transfer, largest sum of equilibrium payoff, and lowest inequality, across all equilibria. Second, autocorrelations hurt incentives: the set of equilibria shrinks as either agent's has more positively autocorrelated income.

In below I present some simulated examples. The goal of the examples to illustrate what happens when the commitment is removed, and what happens with autocorrelations. The top-left panel is the benchmark case with full commitment environment, and shows the set of feasible payoffs. Notice that the set does not vary in autocorrelations. Commitment is removed for the other three panels, which show how the set of equilibrium payoffs (under limited commitment) changes with varying combinations of autocorrelations. Let us consider the gains from risk-sharing, measured as the maximum transfer equilibrium payoff minus the autarky payoff (numbers in charts).

While removing commitment *per se* may not reduce the gains by a lot (100% to 95% when both agents are i.i.d.), autocorrelations bring tremendous losses: the gains drop to 65% as one agent has an autocorrelation type of 0.7, and further to 19% when both agents have positive autocorrelations.

In addition, the gains have *decreasing differences* in autocorrelation types: $95\% - 65\% > 65\% - 19\%$. Hence in the total-gain maximizing matching pattern is negative assortative matching, i.e. two agents with different autocorrelation types (0.5 and 0.7) should be matched together. However, this pattern needs not occur in any matching equilibrium. Such distortions in total-welfare can be a general feature for matching game equilibria. The subsequent sections explore these observations in more details.



Note: parameters used: $u = -e^{-c}, \delta = 0.82, \theta = 0.5$ or 0.7. As a benchmark, the top-left panel is the feasible set, i.e. the set of ex-ante payoffs with full commitment. The other three panels are the set of equilibrium payoffs with varying autocorrelation types (specified in titles). "FB gains" is defined as the MTE payoff minus the autarky payoff.

3.2 Matching shapes the selection of risk-sharing equilibria

How does endogenous matching affect the selection of risk-sharing equilibrium? Will the corresponding maximum transfer equilibrium be selected in each pair? Or will it be some other equilibrium? Thses questions are answered in this section. As illustrated in the following proposition, whether the MTE is selected is determined by whether the pair of agents are of the same type or different types.

Lemma 4 **(Equal payoff for same-type agents)** *In any matching game equilibrium, all agents of a same type receive the same level of payoff. In particular, $U_i \geq U^*(\theta_i, \theta_i)$, i.e. i receives at least the MTE payoff when matched within-type.*

Proposition 2 **(Matching determines selection)** *With endogenous matching, in any matching game equilibrium $\big(m, \{(q_i, q_j)\}\big)$ the selection of risk-sharing equilibria in each pair depends on whether the two agents have the same type. $\forall (i, j), j = m(i)$:*

1. *If $\theta_i \neq \theta_j$ (across-type pair), the agents play an asymmetric equilibrium that favors the less-autocorrelated. Compared to the corresponding maximum symmetric equilibrium, any asymmetric equilibrium results in a reduction in the sum of payoffs, an increase in inequality, and distortions in both agents' transfers.*

2. *If $\theta_i = \theta_j$ (within-type pair), the MTE is played, i.e. $q_i = q_j = q^*(\theta_i, \theta_i)$.*

Matching shapes equilibrium selection because an agent's outside option is determined endogenously in the matching phase. A fisherman, who is less autocorrelated, gets a higher within-type payoff. Such a payoff exceeds the MTE payoff when the fisherman matches with a miner who is more autocorrelated ($U^*(\theta^f, \theta^m) < U^*(\theta^f, \theta^f)$). Therefore, for any across-type match to be present, agents have to play some asymmetric equilibrium that favors the fisherman. In particular, both the fisherman and the miner should get at least their within-type payoffs. If there are more than one such equilibria, which one is selected depends on the relative sizes of the two groups: the selected equilibrium is such that an agent from the relatively smaller group is indifferent between matching across-type and within-type. In sum, the selection of equilibrium is first determined by the relative relative advantages of the types, then the relative population sizes.

The asymmetric treatment results in several distortions, compared to to the MTE that the pair of agents could have chosen without endogenous matching. Both agents choose less transfers, due to the complementarity in agents' incentive structures. An increase in the fisherman's payoff leads to a larger reduction in the miner's.

3.3 Implications for total welfare

In this part I discuss the implications of the above results on total welfare in terms of the the sum of agents' (ex-ante) payoff. Compared to a benchmark in which matching is exogenously controlled by a benevolent social planner who maximizes total welfare, with endogenous matching the outcomes can lead to substantial reductions in total welfare. Either people match suboptimally - so that they match with someone who has similar income patterns, which ends up hurting the population overall - or they match optimally, but then in order to sustain that matching agents end up with very unequal sharing arrangements within a partnership, which in turn lowers overall sharing levels.[24]

With a social planner controlling the matching. Consider a social planner who can assigns pairs of agents to match with each other, yet cannot help to resolve the limited-

[24]Notice that if payoffs were perfectly transferrable, the total welfare would be maximized in equilibria. So the concavity in risk-sharing equilibrium payoff frontier, i.e. the fact that arrangements other than the MTE lead to distortions in the amount of sharing and sum of payoffs, is the source of inefficiency.

commitement issue in their risk-sharing phase.[25] In particular, the set of feasible arrangements for the social planner is $\{(m, (q_i, q_j)_{m(i)=j, i<j}) \mid (q_i, q_j) \in \Gamma(\theta_i, \theta_j), \forall m(i) = j\}$, i.e. any arrangement under the planner's intervention should still have each pair playing some risk-sharing equilibrium.

I now specify a sufficient condition under which equilibrium leads to suboptimal allocations. The condition is $\frac{\partial^2 U^*(\theta_i,\theta_j)}{\partial\theta_i\partial\theta_j} < 0$, a discrete form of which is that the MTE payoff has decreasing differences w.r.t. types. One important class of environments covered by this condition is when agents have CARA utility with any Arrow-Pratt measure of absolute risk aversion.

Proposition **3 (Optimal versus equilibrium arrangements)** *If the MTE payoff has decreasing differences w.r.t. types, equilibrium allocation is never optimal. In particular, the optimal arrangement (that maximizes total welfare) is to have negative assortative matching* [26] *and each pair play their maximum transfer equilibrium; whereas any equilibrium necessarily has either positive assortative matching or non-MTE equilibria in some pairs, hence is never an optimal arrangement.*

Corollary **1** *When agents have CARA utility function $u(c) = const - e^{-\lambda c}, \forall \lambda > 0$, the equilibrium arrangement is never optimal.*[27]

As illustrated in the above results, two sources of suboptimalities arise in equilibrium:

1. Suboptimal matching patterns: even when across-type pairs generates a higher total welfare than the average of two within-type pairs (in terms of the MTE payoffs), the equilibrium matching pattern can be positive assortative.

2. Suboptimal selection of risk-sharing arrangements: every pair of agents with different types is forced to select an equilibrium other than the MTE. Such a selection leads to a reduction in total welfare, an increase in inequality and distortions in both agents' risk-sharing transfers.

A social planner's intervention on matching patterns can correct both suboptimalities in above. The effect on inefficient matching is direct as the planner has full control over the

[25] Otherwise if the social planner can impose transfers, the first-best result with $q^{FB} = 0.5$ can be imposed and agents' payoffs are independent from matching patterns.

[26] When the type space is not continuous, or the second order derivatives do not exist, the condition with cross-partials can be replaced by having strictly increasing differences (see, e.g. Topkis (1998), Milgrom & Roberts (1990)). "Negative assortative matching" means that the agent with the largest type matches with the one with the lowest type, the one with the second-largest type matches with the second lowest, etc. (see, e.g. Becker (1973), Chiappori & Reny (2015).)

[27] See appendix A.2 for explicit results with CARA utilities.

matching. As for the second source: when an intervention impose a matching, no agent has an option choose other partners and hence in an (across-type) pair agents are no longer forced to adopt an asymmetric arrangement. Therefore very pair can choose the MTE, resulting in no distortion due to the selection of equilibria.

Finally, either of the above two cases leads to an inequality among agents of different types. Those agents who face more correlated income expose to more risk in consumptions, despite the fact that all agents share the same average income and income variance.

3.4 Matching patterns

As illustrated in the previous section, with endogenous matching every pair agents with different types is forced to select some risk-sharing equilibrium other than their maximum transfer equilibrium (MTE), resulting in distortions in the amounts of risk-sharing as well as total welfare. As a result, the matching pattern is also biased to the extent that agents tend to match with partners that are similar to them in types. This section analyzes the matching pattern in more detail and provides a simple sufficient condition for positive assortative matching to be the unique matching pattern that can be present in equilibrium. The sufficient condition is characterized with two key factors: 1) the relative positions of the risk-sharing equilibrium frontiers (in terms of the MTE payoffs) and 2) the "effectiveness of transfer" in the across-type pair, i.e. the maximum rate at which the more autorrelated agent can transfer equilibrium payoff to attact the less autocorrelated.

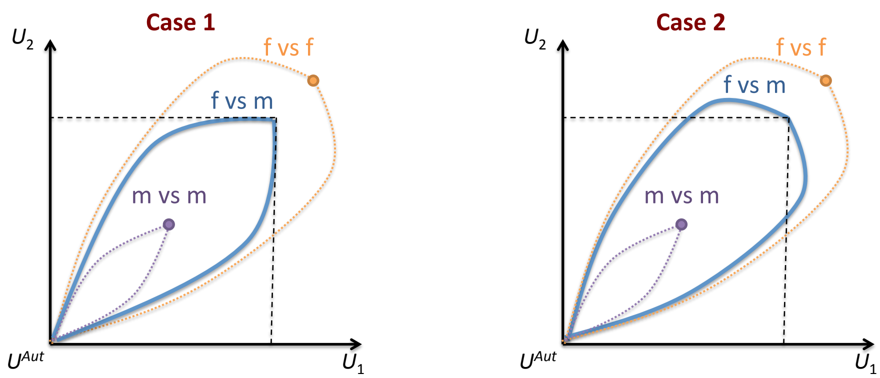
Consider a population with two types of agents: fishermen (f) and miners (m). The fishermen's income is less positively autocorrelated than the miners' ($\theta^f < \theta^m$). When matched within-type, the fishermen gain less than the miners do, and in this sense I call the fishermen a "more-favored" type, and the miners a "less-favored" type.

Two features of the risk-sharing equilibria lead to a tendency for same-type pairs to appear in a matching equilibrium. First, a monotonicity of equilibrium surplus in types, or in another word, everyone prefers a less-autocorrelated partner (fishman). Second, the difficulty for the miner to attract the fisherman - due to the distortions in the amounts of sharing associated to any asymmetric risk-sharing arrangements.

A fisherman can be attracted by a miner to form a risk-sharing partnership if in such an across-type pair agents can find a risk-sharing equilibrium from which both types get at least as much as they could get from matching with a same-type partner (Lemma 4). Whether the existence of such an equilibrium depends on not only the relative positions of the frontiers, but also their shapes. Two cases may occur for an across-type pair with (θ^f, θ^m) regarding the shape of its Pareto frontier:

- Case 1. There exists a risk-sharing equilibrium that Pareto dominates all others; (such a Pareto dominant equilibrium must be the MTE.)

- Case 2. There exists no Pareto dominant risk-sharing equilibrium.



Case 1 is extreme and positive assortative matching is the unique equilibrium matching pattern: the MTE is the most favourite for both types and no one can get a higher payoff from any other risk-sharing equilibrium. As a result, a miner can never attract a fisherman (recall that $U^*(\theta^f, \theta^m) < U^*(\theta^f, \theta^f)$) and thus in equilibrium everyone matches with a same-type partner.

In Case 2, which matching pattern appears in equilibrium depends on two factors: 1) the relative positions of equilibrium frontiers with different combinations of types, and 2) the shape of the across-type equilibrium frontier. The first factor governs how much payoff is needed to attract a fishman, and how much payoff a miner would like to give up, and the second factor determines the effectiveness of "transferring" payoffs from the miner to the fisherman.

In below I present a sufficient condition for positive assortative matching, based on the two factors in above. In particular, the "effectiveness of transfer" for an across-type pair is the maximum rate at which a miner can transfer equilibrium payoff to a fisherman along the equilibrium payoff frontier. Formally:

Definition 4 (Effectiveness of transfer) *The effectiveness of transfer for an across-type pair is defined as* $\gamma(\theta^f, \theta^m) \equiv \sup_{U^m < U^*, (U^f, U^m) \in \mathbb{U}} -\frac{\mathbf{d}U^f}{\mathbf{d}U^m}$.

Due to the concavity of the equilibrium payoff frontier, $\gamma(\theta^f, \theta^m)$ is actually the limit of the rate $\frac{\mathbf{d}U^f}{\mathbf{d}U^m}$ as $U^m \to U^*$ (from below). That is, the rate is maximized at the maximum transfer equilibrium (MTE), and decreases as the risk-sharing arrangement is further away from the MTE.

Recall that when the (ex-ante) utilities are perfectly transferable, positive assortative matching occurs when the surplus has increasing differences in agents' types.[28] There is a

[28] See, e.g. Becker (1973).

corresponding sufficient condition in our environment, in terms of the MTE payoffs:

$$\frac{U^*(\theta^f,\theta^f) - U^*(\theta^f,\theta^m)}{U^*(\theta^f,\theta^m) - U^*(\theta^m,\theta^m)} \geq \gamma(\theta^f,\theta^m), \tag{5}$$

in which the effectiveness of transfer is bounded above: $\gamma(\theta^f,\theta^m) < \frac{u'(1-q^*(\theta^f,\theta^m))}{u'(q^*(\theta^f,\theta^m))} < 1$. To be more specific, $\gamma(\theta^f,\theta^m) \equiv \frac{H}{1-H}\left[\frac{u'(1-q^*)}{u'(q^*)} - \frac{u'(q^*)}{u'(1-q^*)}\right]$.[29]

Proposition 4 **(Sufficient condition for positive assortative matching)** *When* (5) *is satisfied, there is an unique matching game equilibrium in terms of the outcomes, in which*

- *All matches are within-type:* $\theta_i = \theta_j, \forall i, \forall j = m(i)$*;*
- *All pairs play maximum transfer equilibria:* $q_i = q^*(\theta_i,\theta_i), \forall i$*.*

Notice that this Proposition covers Case 1 as a special case: that the MTE is Pareto dominant if and only if $\gamma(\theta^f,\theta^m) \leq 0$ and hence condition (5) is always satisfied. In particular, the expression of γ implies that Case 1 occurs if and only if

$$\frac{u'(1-q^*(\theta^f,\theta^m))}{u'(q^*(\theta^f,\theta^m))} \leq \sqrt{H(\theta^f,\theta^m)}, \tag{6}$$

which therefore is also a sufficient condition for positive assortative matching.

4 Within-Type Common Shocks

In practice, one major source of inefficiency for sharing risk with similar others[30] is that agents who are similar to each other tend to face common shocks to their income. For instance, stormy weather is likely to keep all fishermen in the area staying at home so that they all get low income on that day. Such common shocks reduce the opportunities of risk-sharing among similar agents. This section explicitly models such common shocks.

In particular, agents of the same type are affected by common income shocks. The welfare effects of common shocks are two-fold, depending their frequencies. With less-frequent common shocks, within-type pairs can still be present, resulting in larger inefficiency in equilibrium outcomes. In contrast, frequent-enough common shocks may improve total welfare through changing the matching pattern as well as allowing for the selection of less unequal (i.e. closer to the MTE) risk-sharing arrangements.

[29] Derivation of this appears in the Appendix.

[30] Recall that in our model similar agents are those who are in the same occupation, so that they also have the same autocorrelation type.

Common shocks. I model (type-specific) common shocks as income shocks that, if occur, overwhelm individual shocks for all agents of that type. In particular, for each type θ, a common shock $s^t(\theta)$ occurs with probability of η, equally likely to be 1 or 0, and does not occur (*null*) with a probability of $1-\eta$. An agent i's income is determined by the common shock if $s^t(\theta_i) = 1$ or 0, and determined by his/her individual state s_i^t if the common shock does not occur.

Agents of different types still face independent income flows. This can be viewed as an extreme version of that the income correlation (across agents') is much larger within a occupation than across occupations. In addition, I assume that for each type the (non-null) common shocks share the same autocorrelation as the individual agents' income flow, i.e. $\Pr(s^t(\theta) = s^{t-1}(\theta) \mid s^{t-1}(\theta) = 1 \text{ or } 0) = \theta$. Under this assumption, one can verify that the likelihood of common shocks η has no effect on the set of equilibrium (payoff) for across-type matches.

Common shocks reduce equilibrium payoffs for a within-type pair, since there is less chance for risk-sharing (transfer), which is possible with an ex-ante frequency of $\frac{1-\eta}{2}$ when the common shock does not occur. This also results in an indirect effect through incentives: lower amounts of transfers can be supported in equilibrium. As the likelihood of common shocks increases, the equilibrium payoff frontier moves further towards the origin. This observation is summarized in the following Lemma.

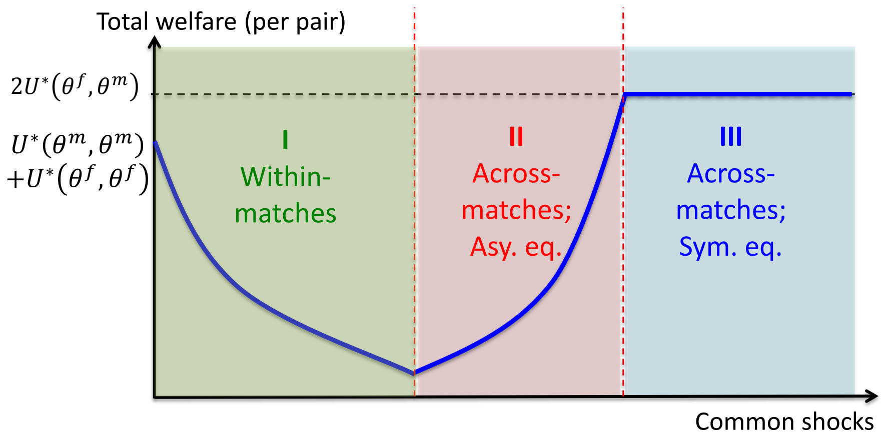
LEMMA **5** *$U^*(\theta_i, \theta_j \mid \eta)$ decreases in η if $\theta_i = \theta_j$, and remains constant if $\theta_i \neq \theta_j$.*

Effects of common shocks on total welfare. For within-type pairs, common shocks result in a reduction of total payoff and hence positive assortative matching can be more inefficient.

However, a larger likelihood in common shocks need not reduce the total welfare, because common shocks can affect the matching patterns and/or the selection of risk-sharing equilibrium in across-type pairs.

The welfare effects of common shocks can be illustrated by the following chart. On the horizontal axis is the likelihood of common shocks η, and on the vertical axis comes the per-pair welfare.

This chart captures the case in which there is no Pareto dominant risk-sharing equilibrium in $\Gamma(\theta^m, \theta^f \mid \eta = 0)$ (Case 2 in section 3.4), so that asymmetric risk-sharing arrangement can occur in the across-type pair. Also it assumes that the equilibrium matching pattern is positive assortative matching at $\eta = 0$.



There are three regimes depending on the likelihood of common shocks η. There are two thresholds of η: $\underline{\eta}$ s.t. both across-type and within-type pairs can occur in equilibrium (and agents are indifferent between the two.) $\overline{\eta}$ s.t. $U^*(\theta^f,\theta^f \mid \overline{\eta}) = U^*(\theta^m,\theta^f \mid \overline{\eta})$.

I $(\eta < \underline{\eta})$: within-type pairs still occur in equilibrium and common shocks reduce welfare.

II $(\underline{\eta} < \eta < \overline{\eta})$: across-pairs occur in equilibrium. In this regime, an increase in η reduces inequality and increase total welfare, as it decreases the more-favored type's "outside option" $U^*(\theta^f,\theta^f \mid \eta)$ and hence an equilibrium closer to the MTE can be selected.

III $(\eta > \overline{\eta})$: agents' payoff from a within-type is too low and the more-favored type would rather stays in an across-type pair even if the MTE is played. In this regime, an increase in η has no further welfare effects.

It is not surprise that common shocks can reduce total welfare. However, (as in regimes II and III) common shocks can also improve total welfare, due to their impacts on the equilibrium matching pattern.

Furthermore, when $U^*(\theta^m,\theta^m) + U^*(\theta^f,\theta^f) < 2U^*(\theta^m,\theta^f)$ at $\eta = 0$,[31] an environment with common shocks may result in a lower inequality (in terms of risk exposure) and a larger total welfare than its counterparty without common shocks. A large likelihood of common shocks may help to correct both suboptimal matching pattern and suboptimal selection of risk-sharing equilibrium.

What if the parameter assumptions made for the above chart does not hold? If the MTE Pareto dominates other risk-sharing equilibria in the across-type pair (Case 1 in section 3.4), then regime II disappears as no other risk-sharing equilibrium can occur in a matching

[31]Recall that a sufficient condition for this is that agents have CARA utilities and face non-negatively autocorrelated income (see subsection 3.3), so that with a social planner who imposes interventions on matching, the optimal arrangement is to have negatively assortative matching (and all pairs play the maximum symmtric equilibrium).

equilibrium, and at $\eta = \overline{\eta}$ the total welfare jumps to its level in regime III. Else if at $\eta = 0$ we already have negative assortative matching, then regime I disappears and common shocks only benefit total welfare.

5 Alternative Heterogeneities

This paper has focused on the heterogeneity of agents' income autocorrelations. This section shows that the machineary developed also applies to various other heterogeneities that can be relevant in practice. Main results presented in previous sections are extended correspondingly. I present three examples:

- heterogeneous time preferences;
- heterogeneous wealth levels (i.e. averages in income);
- heterogeneous income variances.

The basic comparative statics, i.e. the set of equilibria changes monotonically in agents' types, apply to all of the above three examples. Parallel to the fact that everyone likes a less-autocorrelated partner (Proposition 1), with the alternative heterogeneities: everyone likes a more patient partner; everyone likes a wealthier partner; and everyone likes a partner with more variable income.

The lessons on the interplay between matching and risk sharing also apply. In every pair there is a risk-sharing equilibrium in which agents adopt maximum self-enforcing transfers (no need to be symmetric). It is also the equilibrium that maximizes the sum of the two agents' payoffs. With endogenous matching, however, that maximum transfer equilibrium cannot occur when a pair of agents have different types: for the more favored agent, the payoff from such an equilibrium is less than his/her outside option (when matched within-type). As a result, for an across-type pair to occur, the less favored should try harder to attract the more favored. Such an attempt leads to a distortions in both agents' transfers and a reduction in total welfare, due to the complementarity in agents' incentive structures. Finally, the above incentive issues again bias the matching process.

I provide the formal discussions for the alternative heterogeneities in turn. To highlight the effect of each heterogeneity, I assume that agents differ only in the heterogeneity that is currently considered and are equal in other dimensions. In particular, when discussing any of the above, agents are no longer different in autocorrelations: for simplicity, assume that the income shocks are i.i.d. over time and independent across agents. I again consider semi-Markovian strategies in the risk sharing phase.

5.1 Heterogeneous time preferences

There are two types of agents: the more patient and the less patient, with discount factors $\delta^h > \delta^l$ respectively. Agents' income shocks are such that $s_i^t = 0$ or 1, equally likely, i.i.d. over time and independent across agents.

Consider two agents, 1 and 2, that are in a pair. Similar to Lemma 1, the set of risk sharing equilibria $\Gamma(\delta_1, \delta_2)$ equals to the set of self-enforcing semi-Markovian strategies profiles, and is characterized by the following:

$$\frac{u(1) - u(1 - q_1^*)}{u(q_2^*) - u(0)} \leq H(\delta_1);$$

and

$$\frac{u(1) - u(1 - q_2^*)}{u(q_1^*) - u(0)} \leq H(\delta_2);$$

in which $H(\delta_i) \equiv \left(\frac{\delta_i}{4(1-\delta_i)}\right) / \left(\frac{\delta_i}{4(1-\delta_i)} + 1\right)$ increases in δ_i.

The maximun transfer equilibrium (MTE), $\big(q_1^*(\delta_1, \delta_2), q_2^*(\delta_1, \delta_2)\big)$, is the (joint) solution to the above two (self-enforcing) conditions with equalities. By construction $\big(q_1^*(\delta_1, \delta_2), q_2^*(\delta_1, \delta_2)\big)$ is a risk-sharing equilibrium. In addition, due to the complementarity in incentive structures, $q_i^*(\delta_1, \delta_2)$ is the maximum amount of transfer that can be chosen by agent i in any (semi-Markovian) equilibrium:

$$(q_1, q_2) \in \Gamma(\delta_1, \delta_2), (q_1, q_2) \neq \big(q_1^*(\delta_1, \delta_2), q_2^*(\delta_1, \delta_2)\big) \implies q_i < q_i^*(\delta_1, \delta_2),\ i = 1, 2 \tag{7}$$

Let $U_i^*(\delta_1, \delta_2)$ be agent i's payoff in this equilibrium. Parallel to Lemma 3, $U_1^*(\delta_1, \delta_2) + U_2^*(\delta_1, \delta_2)$ maximizes the sum of (ex-ante) payoff that can be achieved among any risk-sharing equilibrium with a pair of types (δ_1, δ_2).[32]

When either agent becomes more patient, the set of equilibria expands and the maximum self-enforcing transfers are larger:

$$\Gamma(\delta^l, \delta^l) \subset \Gamma(\delta^l, \delta^h) \subset \Gamma(\delta^h, \delta^h);$$

$$q_i^*(\delta^l, \delta^l) < q_i^*(\delta^l, \delta^h) < q_i^*(\delta^h, \delta^h), \forall i = 1, 2.$$

An agent's ex-ante payoff only depends on the transfers and his/her own type, but not (directly) on other't type: $U_i = \frac{1}{4(1-\delta_i)} \left[u(q_{-i}) + u(1 - q_i) + u(1) + u(0)\right], \forall i = 1, 2$. Therefore, the monotonicity of the set of equilibrium transfers implies a monotonicity of agents' payoffs:

$$U_i^*(\delta^l, \delta^l) < U_i^*(\delta^l, \delta^h) < U_i^*(\delta^h, \delta^h), \forall i = 1, 2$$

[32] Again we assume $q_i^*(\delta_1, \delta_2) \leq q^{FB} = 0.5, \forall i, \delta_1, \delta_2$, i.e. the first best level of transfer is not sustainable in any equilibrium. A similar assumption is made also in subsections 5.2 and 5.3.

With endogenous matching, $U_i \geq U_i^*(\delta_i, \delta_i)$, i.e. every agent receives a payoff that is at least her "outside option" from a within-type match. This, together with the monotoncity of payoffs, implies that in every pair agents with different types has to select some a risk-sharing equilibrium other than the MTE. The selection leads to distortions in both agents' transfers, and a reduction in total welfare.

PROPOSITION **5 (Risk-sharing in across-type pairs)** *When an across-type pair with* (δ^l, δ^h) *occurs in equilibrium, agents have to select a risk-sharing arrangement* (q_1, q_2) *such that*

$$q_2 < q_1 < q_1^*(\delta^l, \delta^h) < q_2^*(\delta^l, \delta^h). \tag{8}$$

Taking into account of endogenous matching reverses the prediction of "who makes more transfers". In the maximum transfer equilibrium, it is the more patient agent who has incentives to transfer more. In contrast, when endogenize matching is accounted for, the opposite answer applies to the risk sharing equilibrium that is selected: the less patient agent transfers more to (than he/she receives from) the more patient. In addition to this fact, both transfers in the selected equilibrium is below the lowest amount in the maximum transfer equilibrium. Both observations generalize to the situations with other heterogenities (see 5.2 and 5.3).

Finally, some words about the matching patterns. Parallel to (5), a sufficient condition for positive assortative matching to be the only equilibrium matching pattern is

$$\frac{U_2^*(\delta^h,\delta^h) - U_2^*(\delta^l,\delta^h)}{U_1^*(\delta^l,\delta^h) - U_1^*(\delta^l,\delta^l)} \geq \gamma_1(\delta^l, \delta^h), \tag{9}$$

in which $\gamma_1(\delta^l, \delta^h) < \frac{u'(1-q_1^*(\delta^l,\delta^h))}{u'(q_2^*(\delta^l,\delta^h))} < 1$ is the maximum rate at which the agent who has a less favored type can "transfer" equilibrium payoff to her partner.

5.2 Heterogeneous wealth levels

In this part agents are heterogeneous in income levels. As will be illustrated soon, ignoring matching the wealthier agents are the ones who afford to share more money; whereas when matching is accounted for, when a wealthier agent is matched with a poorer agent, in equilibrium they have to select a risk-sharing arrangement in which the wealthier agent receives *more* from the poor.[33]

Formally, income shocks are such that every agent gets 0 on a bad day, but agents with different types get different amounts of income on a good day. An agent's type is the

[33]This observation is consistent to the empirical finding in Misrut (2008). In particular, using detailed inter-household transfer data from Romania, Misrut (2008) finds that the rich receive more. She adopts this finding to argue that transfers are driven by social norms rather than by altruism. The observation just introduced in this paper provides an alternative explaination for the same empirical pattern.

amount he/she receives on a good day: $s_i^t = 0$ or θ_i, with $\theta_i \in \Theta \subset [\underline{\theta}, \overline{\theta}]$. Again assume that good days and bad days are equally likely, and all income shocks are i.i.d. over time and independent across agents.

For a pair of agents 1 and 2, (q_1, q_2) is a (semi-Markovian) risk-sharing equilibrium if and only if

$$\frac{u(\theta_i) - u(\theta_i - q_i)}{u(q_{-i}) - u(0)} \leq H, \forall i = 1, 2$$

Risk aversion implies that $u(\theta_i) - u(\theta_i - q_i)$ strictly decreases in θ_i: for any amout of transfer, it is less costly when the agent is wealtier. Therefore, any transfers (q_1, q_2) that are self-enforcing for a pair of less wealthy agents, they are also self-enforcing for a pair of wealthier agents:

$$(\theta_i, \theta_j) \geq (\theta_i', \theta_j') \implies \Gamma(\theta_i', \theta_j') \subset \Gamma(\theta_i, \theta_j).$$

The above monotonicity result implies results parallel to subsection 5.1: including the selection of equilibrium and its welfare consequences, as well as the condition for matching patterns.

One feature in equilibrium selection is worth noting: parallel to Proposition 5, in every across-type pair with $\theta_1 > \theta_2$, the maximum transfer equilibrium has $q_1^*(\theta_1, \theta_2) > q_2^*(\theta_1, \theta_2)$, since the wealthier agent is afford to transfer more. However, with endogenous matching, agents are forced to select some equilibrium (q_1, q_2) such that $q_1 < q_2 < q_2^*(\theta_1, \theta_2)$. That is, the wealthier agent receives *more* from his/her poorer partner!

5.3 Heterogeneous income variances

In this part I consider the case in which agents share the same average income but may face different variances in income. The analysis presented in this part provides an alternative perspective to the empirical evidence that individuals/households self-select into different risk pools (see, e.g. Ghatak (2000)).

In particular, agent i gets $s_i^t = 0.5 - \theta_i$ on a bad day, and $0.5 + \theta_i$ on a good day, for $\theta_i \in [\underline{\theta}, \overline{\theta}] \subset (0, 0.5]$.[34]

For a pair of agents 1 and 2, (q_1, q_2) is a (semi-Markovian) risk-sharing equilibrium if and only if

$$\frac{u(0.5 + \theta_i) - u(0.5 + \theta_i - q_i)}{u(0.5 - \theta_i + q_{-i}) - u(0.5 - \theta_i)} \leq H, \forall i = 1, 2$$

Risk aversion implies that LHS strictly decreases in θ_i, for any (q_1, q_2): any amount of transfers costs less, and benefits more, for an agent that faces a more various income.

[34]We assume that for any agent i, the first-best transfer to him/her, $q^{FB,i} = \theta_i$, is not achievable in any equilibrium when matched with a partner of any type that is feasible.

Therefore we have the comparative statics of the set of equilibria with repect to income variances:

$$(\theta_i, \theta_j) \geq (\theta_i', \theta_j') \implies \Gamma(\theta_i', \theta_j') \subset \Gamma(\theta_i, \theta_j).$$

The above monotonicity result again implies results parallel to subsection 5.1: including the selection of equilibrium and its welfare consequences, as well as the condition for matching patterns.

6 Rematch and Migration

This section relaxes the assumption that matching is once-and-for-all by allowing for rematch. I highlight two results. First, less friction in the rematch process (captured by a shorter waiting) can make *everyone* worse off. This is due to a negative effect of rematch on risk-sharing incentives: the opportunity of rematching results in a smaller cost in breaking the current relationship, and thus reduces agents' incentives to make a transfer. Second, when agents are heterogeneous in their opportunities of rematch (/migration), everyone prefers to share risk with a partner with worse rematch options and consequencely there is a tendency of sorting with respect to the opportunity of rematch.

6.1 More effective rematch can hurt

A more effective rematch process reduces agents' incentives in risk sharing, and this negative effect can be big enough to overwhelm the direct benefits of a reduced waiting time for being rematched. To highlight this insight, it suffices to consider a simple environment in which agents are homogeneous and face i.i.d. income, equally likely to be 1 or 0 in each period. An existing relationship can be broken in two ways: first, each agent leaves the economy at an exogeneous rate of $\eta > 0$ in each period (due to reasons including death, no more need for informal risk-sharing, etc.); and second, any agent can endogenously opt out. An agent who is currently not in any partnership can (re-)match, which is captured by the effective discount factor for waiting (in a reduced form) by $\delta_r \in (0, 1)$, with the subscript r for "rematch". In particular, δ_r decreases when a longer period of waiting is needed, for instance, when it is easier to migrate to a new place and find new partners to share risk. Finally, the new partners cannot observe what happened to the agent in the past, thus cannot tell whether that agent being available is because her partner left or she chose to opt out or.

Since agents are homogeneous in the matching and rematching process, I focus on the case in which agents adopt symmetric and stationary strategies, so that the amount of transfer is always $q \in (0, 0.5)$ whenever a transfer is needed, in all (initial and rematched) partnerships. I will find the largest self-enforcing transfer q, with which the agents' payoffs will be evaluated.

When rematches are available, the present value (in terms of the net gain from autarky) for an agent who is currently in a partnership, $V(q, \delta_r)$, contains two components: 1) the payoff from the current partnership, in which an agent essentially discounts the future with a factor $\delta_\eta \equiv \delta(1-\eta)^2$, taking into account of the probability that both she and her may leave due to exogenous shocks; 2) the value from the next rematch if her partner leaves (exogenously) earlier than she does, the probability of which is $\sum_{\tau=1}^{\infty} \delta^\tau (1-\eta)^\tau (1-\eta)^{\tau-1} \eta = \frac{\eta}{1-\eta} \frac{\delta_\eta}{1-\delta_\eta}$. The two together lead to

$$V(q, \delta_r) = \frac{u(q) - u(0) + u(1-q) - u(1)}{4(1-\delta_\eta)} + \frac{\eta}{1-\eta} \frac{\delta_\eta}{1-\delta_\eta} \delta_r V(q, \delta_r), \tag{10}$$

i.e.

$$V(q, \delta_r) = \left(1 - \frac{\eta}{1-\eta} \frac{\delta_\eta}{1-\delta_\eta} \delta_r\right)^{-1} \cdot \frac{u(q) - u(0) + u(1-q) - u(1)}{4(1-\delta_\eta)}. \tag{11}$$

Easy to see $\frac{\partial U}{\partial \delta_r} > 0, \forall q > 0$: fixing the amount of transfer, everyone becomes strictly better off when expecting a shorter waiting for being rematched, i.e. when the rematch process is more effective.

The (maximum) size of self-enforcing transfer $q(\delta_r)$, however, does vary in δ_r. More effectiveness of rematch results in a larger outside option for anyone to break the current relationship, and consequently only a lower amount of transfers can be sustained. In particular, the self-enforcing constraint becomes:

$$u(1-q) - u(1) + \delta_\eta V(q, \delta_r) \geq \delta_r V(q, \delta_r), \tag{12}$$

in which the RHS is one's outside option from breaking the current relationship.

(11) and (12) implies that the maximum amount of self-enforcing transfer, $q(\delta_r)$, is a solution to

$$\frac{u(1) - u(1-q(\delta_r))}{u(q(\delta_r)) - u(0)} = H(\delta_r), \tag{13}$$

in which $H(\delta_r) \equiv \left(\frac{\delta_\eta - \delta_r}{4(1-\delta_\eta)}\right) / \left(1 - \frac{\eta}{1-\eta} \frac{\delta_\eta}{1-\delta_\eta} \delta_r + \frac{\delta_\eta - \delta_r}{4(1-\delta_\eta)}\right)$.

LEMMA **6 (Better rematch options hurt incentives)** *When δ_r increases, $H(\delta_r)$ decreases and so does the maximum transfer $q(\delta_r)$.*

When the maximum self-enforcing transfer is used, each agent's expected payoff is

$$V(q(\delta_r), \delta_r) = (\delta_\eta - \delta_r)^{-1} (u(1) - u(1 - q(\delta_r))). \tag{14}$$

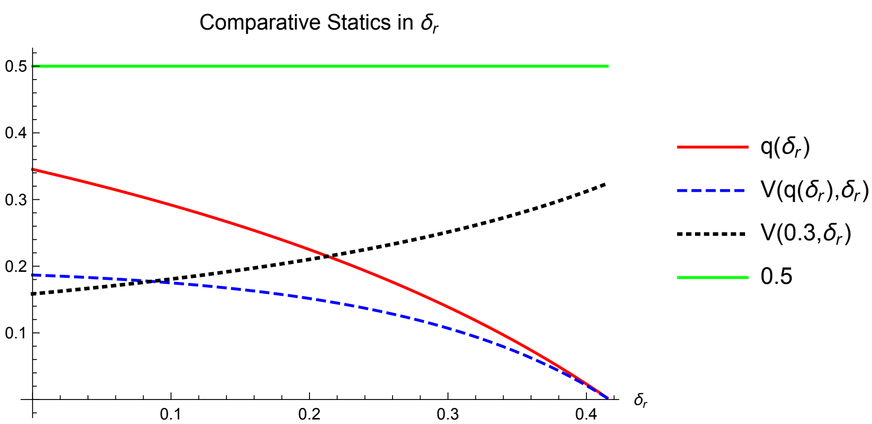
An increase in the effectiveness of rematch market, measured by δ_r, has two effect:

+ direct effect (shorter waiting): an agent's expected payoff goes up, fixing any amount of transfer(s);

– indirect effect (incentives): agents have are less willing to make a transfer, due to a larger outside option from the rematch market.

Due to the negative indirect effect through incentives, a more effective rematch market can make everyone worse off! This is illustrated in the following example.

Example 1 *The figure below plots agents' equilibrium payoff (net gain from risk-sharing, compared to autarky, for someone currently in a partnership), $V(q(\delta_r), \delta_r)$ as a function of effective discount factor for waiting δ_r, for $\eta = 0.05$, $\delta = 0.9$, and CARA utility $u(c) = -e^{-c}$. This is compared to the present value with a fixed favor $q = 0.3$ (so ignoring incentive effects). I also plot how the equilibrium transfer $q(\delta_r)$ varies in δ_r, and compare it to the first-best level* 0.5*.*



As seen in the example, every agent is strictly *worse off* when the rematch market is more effective: the equilibrium payoff is strictly decreasing in g_T. This result is due to the fact that the negative indirect effect (through incentives) overwhelms the positive direct effect (through shorter waiting).

6.2 Heterogeneous opportunities in rematch and migration

This section explores the matching and risk sharing in a population where agents have different rematch opportunities. Agent i's rematch opportunity is again captured by the effective discount factor $\delta_{r,i}$, which decreases in the amount of time she expects to wait before matched with a new risk-sharing partner. Despite of having differential effectiveness in rematch, all agents face the same exogeneous rate of leaving the economy $\eta > 0$.

For any pair of agents 1 and 2, the self-enforcing constraint (induced by (11) and (12) with δ_r replaced by $\delta_{r,i}$'s) is:

$$\frac{u(1) - u(1 - q_i)}{u(q_{-i}) - u(0)} \leq H(\delta_{r,i}), \forall i = 1, 2, \tag{15}$$

recall $H(\delta_{r,i}) \equiv \left(\frac{\delta_\eta - \delta_{r,i}}{4(1-\delta_\eta)}\right) / \left(1 - \frac{\eta}{1-\eta}\frac{\delta_\eta}{1-\delta_\eta}\delta_{r,i} + \frac{\delta_\eta - \delta_{r,i}}{4(1-\delta_\eta)}\right)$.

Better migration options (larger $\delta_{r,i}$) result in a smaller $H(\delta_{r,i})$ (Lemma 6) and hence a less incentive to share risk.

Proposition **6** *With small enough $\eta > 0$ (exogeneous rate of exit), the set of risk-sharing equilibria $\Gamma(\delta_{r,1}, \delta_{r,2})$ and the set of equilibrium payoffs $\mathbb{U}(\delta_{r,1}, \delta_{r,2})$ both shrink as either $\delta_{r,i}$ increases, i.e. either agent has better rematch options.*

$$(\delta_{r,1}, \delta_{r,2}) \geq (\delta'_{r,1}, \delta'_{r,2}) \implies \Gamma(\delta_{r,1}, \delta_{r,2}) \subseteq \Gamma(\delta_{r,1}, \delta_{r,2}),\ \mathbb{U}(\delta_{r,1}, \delta_{r,2}) \subseteq \mathbb{U}(\delta'_{r,1}, \delta'_{r,2})$$

The above proposition implies that the insights presented in the previous parts of the paper also apply to this situation in which agents have heterogeneous opportunities in rematch and migration. In particular, there is a tendency of agents to share risk with those who have similar migration options to them. Also agents with better migration options expose to more risk in consumptions.

7 Conclusion

I present a tractable framework to study the interplay between matching and informal risk sharing. I also explore a novel heterogeneity of agents, who face different autocorrelations in their income flows. Such time patterns are likely to be associated with the agents' occupations.

Autocorrelations do not affect an agents average income, nor the ex-ante frequency of periods in which a pair of agents can help each other. However a high autocorrelation in either agent's income hurts both agents' incentives in risk sharing. Therefore, everyone prefers to share risk with a partner who has a less autocorrelated income.

Endogenous matching determines the selection of risk-sharing equilibria, since an agent's outside option is determined by the payoff from matching with other potential partners. The less autocorrelated have higher outside options, and thus should be favored when matched with an agent who has more autocorrelated income. Such an asymmetric arrangement reduces *both* agents' transfers because of a complementarity in incentive structures: one is willing to make large transfers only when expecting her partner to do so.

In terms of total welfare, either people match suboptimally - so that they match with someone who has similar income patterns, which ends up hurting the population on average and increasing inequality - or they match optimally, but then in order to sustain that matching agents end up with very unequal sharing arrangements within a partnership, which in turn lowers risk sharing levels. In either case there is substantial inequality in terms of risk exposure (in consumption) between two types of agents who face exactly the same mean and variance of income but different income autocorrelations.

This study highlights the importance of, and calls for empirical attention to, time series patterns in income. When ignoring incentives in risk-sharing, autocorrelations do not change risk-sharing potentials between any pair of agents. However, when the incentives are accounted for, autocorrelations are proven to have substantial effects on risk-sharing and matching patterns, and consequently on inequality and total welfare.

As an extension I consider the welfare effects of within-type common shocks, which reduce the gain from risk-sharing for within-type pairs. Such common shocks can reduce or improve total welfare, depending on their frequency. The improvement is because common shocks (when occurring frequently enough) can change matching patterns and reduce the inequality in risk exposure between heterogeneous types.

The framework emphasizes the interaction of matching and play, and applies to general settings. Firstly, it applies to all sources of heterogeneities as long as agents have common preferences over the partners' types. This includes heterogeneities in income levels, income variances, time preferences, and opportunities to migrate and rematch.

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A Appendix

A.1 Proofs omitted from main text

Proof to Lemma 1.

We prove that $H(\theta_1, \theta_2)$ decreases in both arguments. It sufficies to prove that $p^{give,\tau|give}$ increases in both arguments, and $p^{receive,\tau|give}$ decreases in both arguments, $\forall \tau > 0$. In addition, independence between agents implies that $p^{give,\tau|give} = \Pr(s_1^\tau = s_1^0) \times \Pr(s_2^\tau = s_2^0)$, and $p^{receive,\tau|give} = \Pr(s_1^\tau \neq s_1^0) \times \Pr(s_2^\tau \neq s_2^0)$. Hence it sufficies to prove that $\Pr(s_i^\tau = s_i^0)$ increases in θ_i, $\forall i, \tau$.

We prove the above by mathematical induction. For $\tau = 1$, $\Pr(s_i^\tau = s_i^0) = \theta_i$ increases in θ_i. For $\tau \geq 1$, assume that $\Pr(s_i^\tau = s_i^0)$ increases in θ_i; which implies that $\Pr(s_i^\tau = s_i^0) - \Pr(s_i^\tau \neq s_i^0)$ increases in θ_i, since the two terms add up to 1. Now we prove the property for $\tau + 1$. We have

$$\Pr(s_i^{\tau+1} = s_i^0) = \Pr(s_i^\tau = s_i^0) \times \theta_i + \Pr(s_i^\tau \neq s_i^0) \times (1 - \theta_i),$$

and

$$\Pr(s_i^{\tau+1} \neq s_i^0) = \Pr(s_i^\tau \neq s_i^0) \times \theta_i + \Pr(s_i^\tau = s_i^0) \times (1 - \theta_i).$$

Hence $\Pr(s_i^{\tau+1} = s_i^0) - \Pr(s_i^{\tau+1} \neq s_i^0) = (\Pr(s_i^\tau = s_i^0) - \Pr(s_i^\tau \neq s_i^0))(2\theta_i - 1)$ increases in θ_i (recall that $\theta_i \geq 0.5$). Since the two terms in LHS add up to 1, we have $\Pr(s_i^{\tau+1} = s_i^0)$ increases in θ_i. ∎

Proof to Lemma 3.

First prove #3: Assume $\exists(\widetilde{q}_i, \widetilde{q}_j)$ that is self-enforcing such that $\widetilde{q}_i > q^*$. I aim to lead to a contradiction by constructing a symmetric equilibrium with transfers larger than q^* .

Step 1: $\widetilde{q}_i, \widetilde{q}_j > q^*$, otherwise (if $\widetilde{q}_j \leq q^*$) $\frac{u(1)-u(1-\widetilde{q}_i)}{u(\widetilde{q}_j)-u(0)} > \frac{u(1)-u(1-q^*)}{u(q^*)-u(0)} = H(\theta_i, \theta_j)$ violating the self-enforcing constraint.

Step 2: Let $\underset{\sim}{q} = \min\{\widetilde{q}_i, \widetilde{q}_j\}$. $(\widetilde{q}_i, \widetilde{q}_j) > (q^*, q^*)$ is self-enforcing implies that $(\underset{\sim}{q}, \underset{\sim}{q})$ is self-enforcing (due to the symmetry of the self-enforcing constraint) and hence is a (symmtric) equilibrium. Contradiction. Finally, there is no self-enforcing pair (q_i, q_j) with $q_i = q^*$ and $q_j < q^*$, due to the binding self-enforcing constraint at (q^*, q^*). This concludes the proof to #3.

For #1: Maximizing the sum of the ex-ante payoffs is to maximize $u(q_i) + u(1 - q_i) + u(q_j) + u(1 - q_j)$. Notice that $u(q_i) + u(1 - q_i) < u(q^*) + u(1 - q^*), \forall q_i < q^* < 0.5$. Hence (q^*, q^*) maximizes the sum of the ex-ante payoffs among all $(q_i, q_j) \leq (q^*, q^*)$, hence (by #3) among all (semi-markovian) equilibria.

#2 is straightforward since $U_i(q^*, q^*) - U_i(q^*, q^*) = 0$. ∎

Proof to Lemma 4.

Assume not, i.e. $\exists i \neq j$ s.t. $\theta_i = \theta_j$ and $U_i < U_j$. Two cases are possible:

1. if i and j are not matched: i and $m(j)$ can form a blocking pair. $\rightarrow\leftarrow$

2. If i and j are matched: pick another agent k with the same type. It must be the case that k's payoff differs from either i's or j's. Thus we are back to case 1. Finally, if $U_i < U^*(\theta_i, \theta_i)$, then find $j \neq i$ s.t. $\theta_i = \theta_j$. The above results imply $U_j < U_i < U^*(\theta_i, \theta_j)$, and thus i and j can form a blocking pair. $\rightarrow\leftarrow$ ▮

Proof to Proposition 2.

#2 is an immediate corollary to Lemma 4 (by the definition of maximum transfer equilibrium).

For #1: the less-autocorrelated agent, i, gets $U_i(q_i, q_j) \geq U^*(\theta_i, \theta_i) > U^*(\theta_i, \theta_j)$ in which the latter inequality follows the comparative statics for U^*. As a result, the other agent $j = m(i)$ gets less than the maximum symmetric payoff. ▮

Derivation to the bound for $\gamma(\theta^f, \theta^m)$.

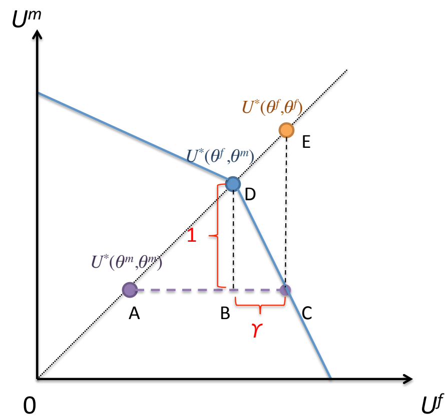
Let $a_i \equiv u(q_i) - u(0)$, $a^* = q^*(\theta^f, \theta^m)$, $v(a_i) \equiv u(1) - u(1 - q_i)$, and $H \equiv H(\theta^f, \theta^m)$ (defined in equation (3)). Also the ex-ante payoff can be re-normalized as $U_i = a_i - v(a_j)$. Suppose agent 1 has θ^f and agent 2 has θ^m. With these notations, the relationship of U_1 and U_2 along the equilibrium payoff frontier can be captured as

$$\begin{array}{rlll} \max_{a_1,a_2} & U_2 & = a_1 - v(a_2) & \\ \text{s.t.} & v(a_1) - Ha_2 & \leq 0 & (IC_1) \\ & v(a_2) - Ha_1 & \leq 0 & (IC_2) \\ & a_2 - v(a_1) & \geq U_1 & \end{array}$$

We need to consider the region with $U_2 > U_1$, in which (IC_1) is binding, i.e. $a_2'(a_1) = H^{-1}v'(a_1)$. Therefore, $\frac{\mathbf{d}U_2(a_1,a_2(a_1))/da_1}{\mathbf{d}U_1(a_1,a_2(a_1))/da_1} = \frac{1-v'(a_2)a_2'(a_1)}{a_2'(a_1)-v'(a_1)}$. The above two equations together imply $\lim_{U_1 \to_- U^*} -\frac{\mathbf{d}U_2}{\mathbf{d}U_1} = \frac{v'(a^*)-\frac{H}{v'(a^*)}}{1-H} = v'(a^*) + \left(v'(a^*) - \frac{1}{v'(a^*)}\right)\frac{H}{1-H}$. Thus $\frac{\mathbf{d}U_2}{\mathbf{d}U_1} < v'(a^*) = \frac{u'(1-q^*)}{u'(q^*)} < 1$ since $v'(a^*) - \frac{1}{v'(a^*)} < 0$. ▮

Proof to Proposition 4.

The reason that condition (5) is sufficient for positive assortative matching can be illustrated with the following graph:



Points A, E, D represent the maximum symmetric equilibrium payoff for a pair with (θ^f, θ^f), (θ^f, θ^m), and (θ^m, θ^m) respectively. Line DC is along the tangency of the risk-sharing equilibrium payoff frontier (the lower half) at point D. Since the frontier is concave, it lies below line DC. By proposition 2, a necessary condition for a (θ^f, θ^m) pair to occur in equilibrium is that there exists some point in $\mathbb{U}(\theta^m, \theta^f)$ that Pareto dominates $(U(\theta^m, \theta^m), U(\theta^f, \theta^f))$. Therefore, it is necessary to have some point on the line DC that Pareto dominates $(U(\theta^m, \theta^m), U(\theta^f, \theta^f))$. The graph plots the threshold case, in which the point $(U(\theta^m, \theta^m), U(\theta^f, \theta^f))$ is on line DC (I call that point C to save notation). By definition, $\gamma = \frac{|DB|}{|BC|}$. Since AD is along the 45-degree line, $|AB| = |BD|$. Hence $\gamma = \frac{|BC|}{|DB|} = \frac{|BC|}{|AB|} = \frac{|DE|}{|AD|} = \frac{U^*(\theta^m,\theta^m) - U^*(\theta^f,\theta^m)}{U^*(\theta^f,\theta^m) - U^*(\theta^f,\theta^f)}$. By construction, we have (5) is sufficient for only within-type pairs to occur in equilibrium. Finally by proposition 2 again, symmetric risk-sharing equilibria shall be selected by every pair. ∎

Proof to Proposition 5.

$q_1^*(\delta^l, \delta^h) < q_2^*(\delta^l, \delta^h)$ is resulted directly from the definition of maximum transfer equilibrium and the self-enforcing constraints: note that $H(\delta^h) > H(\delta^l)$ and hence the δ^h agent has a more relaxed incentive constraint.

We now prove $q_1 > q_2$ for any $(q_1, q_2) \in \Gamma(\delta^l, \delta^h)$ that can be selected in an across-type pair in any matching game equilibrium. Assume not, i.e. $q_1 \leq q_2$, then $U_2(q_1, q_2) \leq U_2(q_2, q_2) < U_2(q_2^*(\delta^l, \delta^h), q_2^*(\delta^l, \delta^h)) \leq U_2(q_2^*(\delta^h, \delta^h), q_2^*(\delta^h, \delta^h))$. That is, in such an equilibrium the more favored type gets a payoff below his/her outside option. This contradicts with Lemma 4.

Finally, the above, together with $q_1^*(\delta^l, \delta^h) > q_1$ completes the proof. ∎

Proof to Lemma 6.

$q(\delta_r)$ increases in $H(\delta_r)$, hence it suffices to show that $H(\delta_r)$ decreases in δ_r. Recall $H(\delta_r) \equiv \left(\frac{\delta_\eta - \delta_r}{4(1-\delta_\eta)}\right) / \left(1 - \frac{\eta}{1-\eta}\frac{\delta_\eta}{1-\delta_\eta}\delta_r + \frac{\delta_\eta - \delta_r}{4(1-\delta_\eta)}\right)$. As δ_r increases, the numerator decreases. The denominator decreases in δ_r: the coefficient of δ_r is written as $-\frac{1-\eta-4\eta\delta_\eta}{4(1-\eta)(1-\delta_\eta)} = -\frac{1-4\eta(1-\eta)\delta}{4(1-\delta_\eta)} \leq 0$, in which the second equality follows $\delta_\eta \equiv \delta(1-\eta)^2$. ∎

A.2 More detail with CARA utilities

This part explores the case in which agents have CARA utility function $u(c) = 1 - e^{-\lambda c}$, $\lambda > 0$. For comparative statics with respect to income levels, let each agent's income levels be $\overline{y}$ and $\underline{y}$, such that $\overline{y} > \underline{y} > 0$. The amount of transfer is correspondingly $q \in [0, \frac{\overline{y}-\underline{y}}{2}]$. In this case, we have the benefit/cost at transferring q in terms of payoff are:

$$u(\underline{y} + q) - u(\underline{y}) = e^{-\lambda(\underline{y}+q)}(e^{\lambda q} - 1);$$

$$u(\overline{y}) - u(\overline{y} - q) = e^{-\lambda \overline{y}}(e^{\lambda q} - 1).$$

Therefore, the self-enforcing constraint (3) between a pair of agents 1 and 2 becomes

$$\frac{u(\overline{y}) - u(\overline{y} - q_i)}{u(\underline{y} + q_{-i}) - u(\underline{y})} = \frac{e^{\lambda q_i} - 1}{e^{\lambda q_{-i}} - 1} e^{-\lambda(\overline{y}-\underline{y}-q_{-i})} \leq H(\theta_1, \theta_2), \forall i = 1, 2, \tag{16}$$

recall $H(\theta_1, \theta_2) \equiv \frac{\sum_\tau \delta^\tau p^{receive,\tau|give}}{\sum_\tau \delta^\tau p^{give,\tau|give} + 1}$ strictly decreases in θ's.

In particular, the transfer in maximum transfer equilibrium (MTE), $q^*(\theta_1, \theta_2)$, solves the following equation

$$e^{-\lambda(\overline{y}-\underline{y}-q^*(\theta_1,\theta_2))} = H(\theta_1, \theta_2). \tag{17}$$

Thus the MTE payoff

$$\begin{aligned} U^*(\theta_1, \theta_2) &= \frac{1}{4(1-\delta)} \left[u(\underline{y} + q^*) + u(\overline{y} - q^*) + u(\overline{y}) + u(\underline{y}) \right] \\ &= \frac{1}{4(1-\delta)} \left[4 - e^{-\lambda \underline{y}} H(\theta_1, \theta_2) - \frac{e^{-\lambda \overline{y}}}{H(\theta_1, \theta_2)} - e^{-\lambda \underline{y}} - e^{-\lambda \overline{y}} \right] \end{aligned} \tag{18}$$

One observation is that the MTE payoff, $U^*(\theta_1, \theta_2)$, has decreasing differences in autocorrelation types. In particular, $\frac{\partial^2 U^*(\theta_1,\theta_2)}{\partial\theta_1 \partial\theta_2} = \Big((1 - H)(e^{-\lambda \underline{y}} - \frac{e^{-\lambda \overline{y}}}{H}) \Big)_{12} = (-e^{-\lambda \underline{y}} + e^{-\lambda \overline{y}} H^{-2}) H_{12} - 2e^{-\lambda \overline{y}} H^{-3} H_1 H_2 < 1$, in which $H_i \equiv \frac{\partial H}{\partial \theta_i}$ and $H_{12} \equiv \frac{\partial^2 H}{\partial \theta_1 \partial \theta_2}$, and one can verify $H_{12} > 0$ and $(1 - e^{\lambda(\overline{y}-\underline{y})} H^2) \frac{H H_{12}}{2 H_1 H_2} < 1$. Hence if ex-ante payoff is perfectly transferable between agents, then negative assortative matching will result.

In below we discuss the comparative statics of matching patterns with respect to income levels.

LEMMA 7 *When agents have CARA utilities, the condition for positive assortative matching, (5), is more likely to hold (LHS increases and RHS decreases) when $\overline{y}$ becomes smaller or $\underline{y}$ becomes larger.*

As a result, a policy/shock that increases the low income (e.g. subsidies in bad periods) tends to push the matching patterns more toward positive assortative matching, whereas a policy/shock that increases the high income (e.g. better prices for fish and coal) tends to allow for more across-type pairs.

Finally, with CARA utilities, $\frac{u'(1-q^*(\theta^f,\theta^m))}{u'(q^*(\theta^f,\theta^m))} = -e^{-\lambda(\overline{y}-\underline{y})}$, $\forall(\theta^f,\theta^m)$. Therefore, the necessary and sufficient condition (6) for case 1, i.e. the MTE is Pareto dominant for an across-type pair becomes:

$$-e^{-\lambda(\overline{y}-\underline{y})} \leq \sqrt{H(\theta^f,\theta^m)}, \tag{19}$$