Abstract—We present a model to evaluate the upper limit of the spectral efficiency of optical code-division multiple-access (OCDMA) systems with coherent sources. Phase-encoded and direct-sequence OCDMA systems are evaluated using this model. The results show that a spectral efficiency of $2.24 \times 10^{-2}$ b/s•Hz can be achieved with a maximum bit error rate of $10^{-10}$ in these systems of the number of users. This result demonstrates that the maximum spectral efficiency of OCDMA systems with coherent sources is at least a factor of 5 higher than OCDMA systems with incoherent sources.

Index Terms—Code division multiplexing, communication system performance, encoding, optical noise, pulsed laser.

I. INTRODUCTION

Since the late 1980s, the optical communication technology has adapted to gradually support the cabled telecommunication infrastructure from long-distance systems down to access systems. Since then, the key solution of large-scale systems is based on wavelength-division multiplexed (WDM) channels individually containing aggregated time-division multiplexed (TDM) signals. While WDM/TDM are being considered at smaller-scale for access and local systems, other potential approaches such as optical code-division multiple-access (OCDMA) are currently under investigation.

The spectral efficiency ($\Sigma_{\text{BER}}$) is a fundamental measure of the performance of an optical communication system. It specifies the overall throughput per unit of optical bandwidth associated with a fixed bit error rate (BER)

$$\Sigma_{\text{BER}} = \frac{N_U R}{B_o}$$

(1)

where $N_U$ is the number of simultaneous users emitting at a bit rate $R$ and where $B_o$ is the optical bandwidth occupied by the system. Because of the limited bandwidth resource, the highest performance systems are generally those that achieve the largest spectral efficiency at identical BER. In this sense, the use of WDM/TDM for large-scale systems is effective because the aggregation of a large number of users, combined with an electronic provisioning, feed the WDM channels with relatively stable data flow. However, the use of WDM and/or TDM for access communication may be of limited efficiency because of the low bit rate required per user, the sporadic data flow, and the electronic processing that currently enables TDM. OCDMA systems are attractive for the access networks. These systems provide to each user an access to the network without need for traffic management or system synchronization. Also, every OCDMA signal is all-optically integrated to the network without requiring electronic multiplexing.

Within the OCDMA category, systems with coherent sources have been discussed in [1]–[3] and theoretically demonstrate better BER results than OCDMA systems with incoherent sources [4]. In this paper, we evaluate the spectral efficiency of bipolar phase-encoded (PE) OCDMA and direct-sequence (DS) OCDMA systems. We show that the upper limit of the spectral efficiency for both optimized systems turn into the same mathematical expressions. We develop analytical equations and perform numerical simulations for the spectral efficiency of both bipolar OCDMA systems. In the work described herein, our goal is to highlight the fundamental limitation of OCDMA systems with coherent sources. This limitation arises from the multiple access interference (MAI) of simultaneous interferers. From a quantitative expression of the MAI, we evaluate the best spectral efficiency physically attainable even though major technological improvements are still required to reach these performances [2], [5]. Finally, because the suppression of spectral components of a coherent source distort its pulse shape, we limit the investigation of OCDMA systems with coherent sources to those that only manipulate the phase, whether in the frequency domain (PE-OCDMA) or in the time domain (DS-OCDMA). Those encoding types do not remove power from the spectral content, and entirely restore the pulse after encoding/decoding.

II. ARCHITECTURE

The architecture of a bipolar PE- and DS-OCDMA system is illustrated in Fig. 1. The OCDMA network considered includes several transmitters (Tr) and receivers (Re) placed in periphery of a star network configuration. A transmitter is composed of an optical source sending ultrashort pulses encoded into either logical 1 or logical 0. Once encoded, a pulse converts into a pseudonoise burst. On the receiver side, the encoded signals pass through two decoders and a pair of balanced photodiodes. The decoders are complementary copies of the encoders and the balanced photodiodes provides a differential signal between both decoded signals. We suppose that the balanced photodiodes and the ensuing electronics have an electrical bandwidth large enough ($\cong B_o$) to resolve the instantaneous power of the incoming signals. Note that in several practical applications, the bandwidth required for the electronics is too large and solutions
using nonlinear optics are actually under investigation to temporally resolve such ultrashort pulses [2], [6], [7]. The temporal resolution provided with a high-bandwidth detection maximizes the extinction ratio between the pulse peak and the noise floor at sampling time.

The encoder and decoder perform different functions for PE or DS systems. The PE encoder and decoder spectrally decompose the incoming light, set a pseudorandom phase shift to individual spectral components and recombine them [2], [8], [9]. The DS encoder and decoder make several copies of the incident pulse, route them with pseudorandom delays and recombine them [10], [11]. In [12], it is found that the superposition of several (~10) independent optical sources results in an instantaneous power having the statistical properties of a thermal source. The same effect is found by randomizing the phase of several spectral component of a pulsed source (like in PE) or by superposing several randomly delayed pulse trains (like in DS). The validity of this approximation depends on the code choice and length. Pseudorandom codes such as binary \( m \)-sequence [8]–[10] and quaternary \( a \)-sequence [11] have been proposed to encode the PE and DS OCDMA systems. In this analysis, \( m \)-sequence codes are considered for both PE and DS systems. The pulsed laser spectrum is taken into account to find the appropriate code length. The spectrum of the mode-locked laser used for the simulations has a flat envelope with a bandwidth \( B_0 \) constructed from a comb-like structure of \( M = B_0/R \) spectral lines spaced by \( R \) [Fig. 2(a)]. In the time domain, the laser output is formed of ultrashort pulses of duration \( 2/B_0 \) emitted at a repetition rate \( R \). The peak pulse power is \( P_{\text{ML}} \) whereas the time averaged optical power of a pulse whether in its original state or once converted into pseudonoise [Fig. 2(b)] is \( \langle P_{\text{ML}} \rangle \). The ratio between these two quantities is given by

\[
\frac{P_{\text{ML}}}{\langle P_{\text{ML}} \rangle} = B_0 \frac{\int_0^{B_0} |E(f)|^2 df}{\int_0^{B_0} |E(f)|^2 df} \equiv M
\]  

(2)

where \( E(f) \) is the electrical field of the laser. The ratio expressed by (2) is taken as the rigorous definition of \( M \). As aforementioned, \( M \) equals the number of spectral lines in the context of lasers with uniform spectra. However, in the context of lasers with nonuniform spectral shapes, \( M \) can be interpreted as a spectral form factor rather than a number of spectral lines. This definition is helpful for the spectral efficiency evaluation of a system with lasers of nonuniform spectra.
By numerical simulation, we find that the condition of pseudo-thermal instantaneous power is achieved using pulsed laser sources with $M > 20$ spectral lines and a code length equal or longer than $M$. With a PE-OCDMA system, a code length equal to $M$ implies that each spectral line is individually phase encoded. Once encoded, a pulse is transformed into pseudonoise with a sinc-squared envelope and a main lobe duration of $2/R$. At a repetition period of $1/R$, the encoded pulses result into a pseudonoise with a flat envelope. With the DS-OCDMA system, codes of length $M$ convert an ultrashort pulse into $M$ ultrashort pulses spreading randomly and superposing into a time interval of $2/R$ (encoding plus decoding). Pseudorandom optical signals are analyzed in the view of stationary random processes that are ergodic in the strict sense.

The encoding of logical 1 as well as logical 0 is supported by a bipolar architecture. This ensures that a properly decoded signal produces an eye diagram with an optimal decision point independent of the number of interferers, in contrast to the unipolar configuration where the optimal decision point is changing rapidly and randomly with the number of instantaneous interferers. At detection, the receiver is composed of two decoders encoded with complementary replicas of the desired user code. The decoder associated with the logical 1 converts a logical 1 from the desired user back into a pulse identical to the one before encoding, and similarly for the decoder associated with the logical 0. In contrast, encoded pulses from interferers remain as pseudonoise after passing through either the decoder or its complement. Note also that a pulse encoded as a logical 1 by the desired user remains as pseudonoise after passing through the logical 0 decoder and vice versa.

III. SYSTEM ANALYSIS

In this section, we build the expressions necessary to evaluate the BER and the spectral efficiency of the system. For this purpose, four distinct cases of interest are considered. These are formed by considering the optical power either at PD1 or PD0 (in Fig. 1) when the desired user is transmitting either a logical 1 or a logical 0. First, the probability density functions (pdfs) at the photodiodes for the four cases of interest are calculated. Then, the pdfs from both PD1 and PD0 are convolved to generate the pdfs after balanced detection. Finally, a BER expression is integrated and leads to an analytical expression for the spectral efficiency. These steps are given in more details in the following paragraphs.

At PD1, $N_U - 1$ interferers are simultaneously sending optical noise while the desired user is sending either a logical 1 decoded as an optical pulse, or either a logical 0 decoded as optical noise. Inversely at PD0, a logical 1 is decoded as optical noise and a logical 0 is decoded as an optical pulse. Therefore, a logical 1 at PD1 or a logical 0 at PD0 result in identical pdfs representing the optical power of a pulse added to noise from interferers. Similarly, a logical 0 at PD1 or a logical 1 at PD0 result in identical pdfs representing the optical noise from the desired user and interferers. The four cases of interest then degenerate into two cases: a bit perceived as an optical pulse or as optical noise from the desired user added to a noise background from $N_U - 1$ interferers.

In the case where a pulse is received simultaneously with $N_U - 1$ units of interfering noise, the pdf of the optical power at the peak value of the pulse is [1], [12]

$$f_{P\text{Pulse}}(P|\text{Pulse}) = \frac{1}{(P_{\text{ML}})(N_U - 1)} \exp \left[ -\frac{(P + P_{\text{ML}})}{(P_{\text{ML}})(N_U - 1)} \right] I_0 \left[ \frac{2\sqrt{PP_{\text{ML}}}}{(P_{\text{ML}})(N_U - 1)} \right] U[P].$$  \hspace{1cm} (3)

where $P$ is the optical power, $P_{\text{ML}}$ is the peak pulse power coming from the desired user, $(P_{\text{ML}})$ is the mean optical power from an interferer, $N_U$ is the number of active users, $I_0$ is the modified Bessel function of the first kind and zero order, and $U[P]$ is the unit step function that is unitary when $P \geq 0$ and zero elsewhere. In the case where the bit from the desired user is decoded as optical noise, the total power is composed only of noise from $N_U$ noisy users ($N_U - 1$ interferers plus the desired user). The resulting pdf of the optical power is a decaying exponential [1], [12]

$$f_{P\text{Noise}}(P|\text{Noise}) = \frac{1}{(P_{\text{ML}})N_U} \exp \left[ -\frac{P}{(P_{\text{ML}})N_U} \right] U[P].$$  \hspace{1cm} (4)

Fig. 3 presents an example of $f_{P\text{Pulse}}(P|\text{Pulse})$ and $f_{P\text{Noise}}(P|\text{Noise})$. Note that (3) and (4) lead to identical expressions if the signal from the desired user is off, i.e., $P_{\text{ML}} = 0$ in (3) and $N_U$ is replaced by $N_U - 1$ in (4).

At the sampling time, the desired user is sending exclusively a logical 1 or a logical 0. Therefore, the pdfs represented by (3) and (4) are complementary on PD1 and PD0. If a pulse is perceived on PD1, then only noise is perceived at PD0 and vice versa. The signal after balanced detection is represented by the optical power difference between a pulse in presence of interference and interference only. The pdf of this optical power difference is expressed by a convolution between $f_{P\text{Pulse}}(P|\text{Pulse})$ and $f_{P\text{Noise}}(-P|\text{Noise})$

$$f_{P\text{Pulse}|-\delta P|}(\delta P|) = f_{P\text{Pulse}}(P|\text{Pulse}) \ast f_{P\text{Noise}}(-P|\text{Noise})$$  \hspace{1cm} (5)

when the desired user is sending a logical 1 perceived as an optical pulse at PD1 and as noise at PD0. $\delta P$ is the differential
optical power between PD1 and PD0. Similarly, when the desired user is sending a logical 0, \( f_{SP0}(\delta P) = f_{SP1}(\Delta P) \). Fig. 4 presents an example of \( f_{SP1}(\delta P) \) and \( f_{SP0}(\Delta P) \). Now, (6) is solved from (3), (4)

\[
f_{SP1}(\delta P) = \frac{1}{P_{ML}^2 N_U(N_U - 1)} \exp \left[ - \frac{P_{ML}N_U - \delta P(N_U - 1)}{P_{ML}N_U(N_U - 1)} \right] \times \int_{\text{Max}[0,\delta P]}^{\infty} \exp \left[ - \frac{P(2N_U - 1)}{P_{ML}N_U(N_U - 1)} \right] \times I_0 \left[ \frac{2\sqrt{P}P_{ML}}{(P_{ML})(N_U - 1)} \right] dP
\]

where \( \text{Max}[0,\delta P] \) expresses the maximum value between 0 and \( \delta P \). The BER is found using

\[
\text{BER} = 0.5 \left[ \int_{-\infty}^{\eta} f_{SP1}(\delta P) d\delta P + \int_{\eta}^{\infty} f_{SP0}(\delta P) d\delta P \right].
\]

The optimal optical power threshold \( \eta \) is calculated by solving

\[
f_{SP1}(\eta|1) = f_{SP0}(\eta|0)
\]

that leads to \( \eta = 0 \) since \( f_{SP0}(\delta P|0) = f_{SP1}(\Delta P|1) \). By symmetry, (7) reduces to

\[
\text{BER} = \int_{-\infty}^{0} f_{SP1}(\delta P|1) d\delta P.
\]

With these observations, and combining (6) into (9), we find using the following equalities: \( \text{Max}[0,\delta P] = 0 \) since the maximum integration boundary of \( \delta P \) in (9) is 0, \( P_{ML} = M(P_{ML}) \)

\[
\int_{-\infty}^{0} \exp \left[ - \frac{\delta P}{P_{ML}N_U} \right] d\delta P = \langle P_{ML} \rangle N_U
\]

and

\[
\int_{0}^{\infty} \exp \left[ - \frac{P(2N_U - 1)}{P_{ML}N_U(N_U - 1)} \right] I_0 \left[ \frac{2\sqrt{P}P_{ML}}{(P_{ML})(N_U - 1)} \right] d\delta P
\]

(11)

(9) reduces to

\[
\text{BER} = \frac{N_U}{2N_U - 1} \exp \left[ - \frac{M}{2N_U - 1} \right].
\]

This expression provides the optimal BER theoretically attainable under the best experimental conditions. The BER decreases with increasing mode number \( M \) and increases with the number of users \( N_U \). Take note that (12) is valid independent of the laser spectral shape. Now considering the special case of lasers with uniform spectral shape (and, therefore, with \( M = B_0/RI \)), the spectral efficiency in (1) equals

\[
\Sigma_{\text{BER}} = \frac{N_U}{M}
\]

and rearranging (12)

\[
\Sigma_{\text{BER}} = \frac{-N_U}{2N_U - 1} \ln \left[ \frac{1}{\frac{2N_U - 1}{N_U \cdot \text{BER}}} \right].
\]

The spectral efficiency is essentially a function of the BER and also a function of the number of users to a limited extent since the dependence with \( N_U \) disappears for \( N_U \gg 1 \).

IV. RESULTS

In Fig. 5 we present simulation results for spectral efficiency as a function of the number of active users for two qualities of service (QoS). For the best QoS, with \( BER = 10^{-10} \), the asymptotic spectral efficiency can be calculated from (14) to be \( \Sigma_{\text{BER}} = 2.24 \times 10^{-2} \) b/s/Hz. Relaxing the QoS to \( BER = 10^{-5} \) (where forward error correction can be used to improve error rate) the asymptotic spectral efficiency doubles to \( \Sigma_{\text{BER}} = 4.62 \times 10^{-2} \) b/s/Hz when \( N_U \gg 1 \). In Fig. 5 we see the
curves are very flat, reflecting the limited dependence on the number of users. A spectral efficiency that is independent of \( N_U \) means that an equally performing optimal design is available independent of the system size. Referring to (13) and \( \Sigma_{\text{BER}} \approx 2.24 \times 10^{-2} \text{ bits/s Hz} \) when \( \text{BER} = 10^{-10} \), one must allow \( \sim 45 \) spectral lines per user \( (= \frac{1}{\Sigma_{\text{BER}}}) \) on every laser source to reach the optimal spectral efficiency.

We now consider the effect of a nonuniform spectral shape of the lasers on the spectral efficiency. From the analysis of systems with uniform spectral shape, a modification in the spectral shape profile affects only the ratio \( P_{\text{ML}}/(P_{\text{ML}}) = M \) in (12). From (12), it is found that the BER can be kept constant for different values of \( M \) by a proper counter-balance of \( N_U \). For instance, considering laser spectrums that are Gaussian with full-width at half maximum equal to half the system bandwidth, it is found from (2) that \( M, 1/N_U, \) and \( \Sigma_{\text{BER}} \) are reduced to 75% of their uniform spectral profile value. Such systems therefore get a maximum spectral efficiency of \( \Sigma_{\text{BER}} \approx 1.68 \times 10^{-2} \text{ bits/s Hz} \).

Finally, we can compare our results with those of OCDMA systems with incoherent sources. In previous papers [13], [14], we demonstrated that the spectral efficiency of the best performing OCDMA system with incoherent sources (instead of coherent sources) is \( \Sigma_{\text{BER}} = 4.20 \times 10^{-3} \text{ bits/s Hz} \) at \( N_U = 10 \) users and \( \text{BER} = 10^{-10} \). The maximum spectral efficiency of these systems decreases as \( N_U \) increases, a second disadvantage as compared to coherent OCDMA systems. The practical implementation challenges of these two approaches and relative cost will determine which system is appropriate for a particular application.

V. DISCUSSION AND CONCLUSION

As an example of system design, to accommodate 100 users \((= N_U)\) requires laser sources with \( M = 4445 \) spectral lines to attain \( \text{BER} = 10^{-10} \) and the optimal spectral efficiency of \( \Sigma_{\text{BER}} = 2.25 \times 10^{-2} \text{ bits/s Hz} \). Typical femtosecond pulsed lasers found commercially achieve such a number of lines (up to \( 10^9 \)) spectrally separated by tens to hundreds of megahertz and spreading over several nanometers. Also of practical interest, in a system using mode-locked lasers with fixed bandwidths, a tradeoff between the desired number of users and their individual bit rate must be performed to attain the optimal spectral efficiency. For instance, lasers with a fixed bandwidth but with variable repetition rate will have their repetition rate adjusted in order to satisfy (13) and (14) simultaneously.

In previous papers [13], [14], we demonstrated that the spectral efficiency of the best performing OCDMA system with incoherent sources (instead of coherent sources) is \( \Sigma_{\text{BER}} = 4.20 \times 10^{-3} \text{ bits/s Hz} \) at \( N_U = 10 \) users and \( \text{BER} = 10^{-10} \). Moreover, the maximum spectral efficiency of systems with incoherent sources is decreasing as \( N_U \) increases. The spectral efficiency of \( \Sigma_{\text{BER}} = 2.24 \times 10^{-2} \text{ bits/s Hz} \) when \( N_U \gg 1 \) for systems with coherent sources is more appealing for two reasons. First, the absolute value of the spectral efficiency with coherent sources is larger by a factor of more than five at low \( N_U \). Second, the spectral efficiency follows a constant asymptote instead of decreasing with \( N_U \), thereby enabling an optimal design at any system size. Therefore, OCDMA systems with coherent sources are the best performing systems when comparing coherent and incoherent sources.

REFERENCES


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