**The unit sample and unit step**

Let’s examine some special signals, first in discrete time, then in continuous time.

**Definition 1.1.** *The discrete time unit step is given by*

*The unit sample or impulse is defined as*

We notice that they are related via the sum relation

Notice the unit sample sifts signals

**Proposition 1.1.** *The unit sample has the “sampling property,” picking off values of signals that it sums against:*

This is true for all signals, implying we can derive various properties, the “summed” and “differenced” versions. Defining , then

Now let’s examine linearity and time invariance.

**Example 1.1.** *Define the system with input x[n] and output y[n]=nx[n]. This system is linear but not time invariant.*

*To see linearity is straightforward, Take linear combinations of inputs and verify outputs are linear combinations. To see the system is not time invariant, define input , then output is for all n. Now shift the input, . But for this .*

**Definition 1.2.** *The continuous time version has a similar form:*

*The differentiated for is the unit impulse:*

**Proposition 1.2.** *The sifting property follows for smooth functions:*

To see this, we can argue loosely swapping the order of the limit according to

In particular, defining x(t)=u(t), then we see the integrated property follows