A Convex Optimization Framework for Constrained Concurrent Motion Control of a Hybrid Redundant Surgical System

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Current Less Invasive Treatment of Osteolysis

- Osteolysis: progressive bone loss
- Wear of the polyethylene liner of the acetabular implant
- Access to lesion area through screw holes of a well-fixed acetabular cup.
- It requires manual removal of osteolysis using rigid tools.
- Less than 50% of the lesion is actually cleaned and grafted.
Osteolysis Treatment Using a Continuum Dexterous Manipulator (CDM)

- **Ortho-snake** is made of two Nitinol tubes with outer diameter of 6 mm and a 4 mm tool channel for inserting different tools.

- Experiments show this CDM is able to explore over 94% of the lesion space.

Proposed Hybrid Redundant Surgical System (6+2 DoF)
Constrained Concurrent control of the Hybrid Redundant System

Assumptions:
1. The whole Flexible region of the CDM is inside the patient’s body
2. No external force is applied to the tip of the CDM

Constraint: Effect of screw holes on Degrees of Freedom
- No mechanical remote center of motion (RCM) for UR5

Literature Review (Redundancy Resolution)

- Literature documents **pseudoinverse-based gradient projection** and **optimization-based** approaches.

- The optimization-based algorithms and especially **constrained least-squares formulation** simplifies implementing equality and inequality constraints.

- Funda et al. [1] and Kapoor et al. [2] have implemented this method to control different medical robots.

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The damped least-squares method (DLM)

- The damped least-squares method (DLM) is a specific type of the least-square formulation
  \[
  \arg\min_{\Delta\theta} ||J(\theta)\Delta\theta - \Delta x||_2^2 + \lambda ||\Delta\theta||_2^2
  \]
  where \( J(\theta) \in \mathbb{R}^{m \times n} \) (\( m < n \)), \( \Delta\theta \in \mathbb{R}^n \), \( \Delta x \in \mathbb{R}^m \), \( \lambda > 0 \) ∈ \( \mathbb{R} \).

- It deals with the singularity avoidance problem.

- It is a trade-off between the accuracy with which the desired end-effector trajectory is followed and the feasibility of the joint velocity.

- The damping parameter \( \lambda \) has a direct effect on the performance.

- Closed-form solution exist in the case of constraint-free problem:
  \[
  \Delta\theta = J^T(JJ^T + \lambda I_m)^{-1}\Delta x
  \]
Goal of the Current Study

- We propose a new algorithm for solving the DLM problem subject to multiple linear equality and/or inequality constraints:

\[
\underset{\Delta \theta}{\text{argmin}} \| J(\theta) \Delta \theta - \Delta x \|^2 + \lambda \| \Delta \theta \|^2
\]

Subject to \( A \Delta \theta \leq b \)

where \( J(\theta) \in \mathbb{R}^{m \times n} \) (\( m < n \)), \( \Delta \theta \in \mathbb{R}^n \), \( \Delta x \in \mathbb{R}^m \), \( \lambda > 0 \in \mathbb{R} \).

- A constrained \( L_2 \)-regularized quadratic minimization problem.

- We implemented a technique of convex optimization called \textit{Alternating Direction Method of Multipliers} (ADMM).

- Ghadimi et al. [1] showed optimally tuned ADMM method can significantly outperform existing alternatives approaches.

- We prove the linear convergence of the proposed method and show it is independent of the choice of DLM damping parameter.

The ADMM algorithm

- The ADMM algorithm solves **structured** convex optimization problems:

$$\begin{align*}
\text{minimize} & \quad f(x) + g(z) \\
\text{subject to} & \quad Ax + Bz = c
\end{align*}$$

where $f$ and $g$ are convex functions, $x \in \mathbb{R}^n$, $z \in \mathbb{R}^m$, $A \in \mathbb{R}^{p \times n}$, $B \in \mathbb{R}^{p \times m}$ and $c \in \mathbb{R}^p$.

- To solve this problem, we form the augmented Lagrangian:

$$L_\rho(x, z, y) = f(x) + g(z) + y^T(Ax + Bz - c) + (\rho/2)\|Ax + Bz - c\|_2^2$$
The ADMM algorithm

- **A sequential minimization** of the $x$ and $z$ variables followed by a dual variable $y$ update can be considered:

  \[
  x_{k+1} = \arg\min_x f(x) + y_k^T(Ax + Bz_k - c) + \frac{\rho}{2}\|Ax + Bz_k - c\|_2^2
  \]
  \[
  z_{k+1} = \arg\min_z g(z) + y_k^T(Ax_{k+1} + Bz - c) + \frac{\rho}{2}\|Ax_{k+1} + Bz - c\|_2^2
  \]
  \[
  y_{k+1} = y_k + \rho(Ax_{k+1} + Bz_{k+1} - c)
  \]

The convergence of the ADMM algorithm was usually defined by the primal $r_{k+1}$ and dual $s_{k+1}$ residuals:

\[
 r_{k+1} = Ax_{k+1} + Bz_{k+1} - c \\
 s_{k+1} = \rho A^TB(z_{k+1} - z_k)
\]
The ADMM algorithm for solving the DLM problem

- Converting the DLM problem to the standard ADMM form

\[
\begin{align*}
\argmin_{\Delta \theta} & \quad \|J(\theta)\Delta \theta - \Delta x\|_2^2 + \lambda \|\Delta \theta\|_2^2 \\
\text{Subject to} & \quad A \Delta \theta \leq b
\end{align*}
\]

\[
\argmin_{\Delta \theta} \frac{1}{2} \|J(\theta)\Delta \theta - \Delta x\|_2^2 + \frac{\lambda}{2} \|\Delta \theta\|_2^2 + I_+(z) \\
\text{Subject to} & \quad A \Delta \theta - b + z = 0
\]

- The associated augmented Lagrangian would be:

\[
L_\rho(\Delta \theta, z, u) = \frac{1}{2} \|J(\theta)\Delta \theta - \Delta x\|_2^2 + \frac{\lambda}{2} \|\Delta \theta\|_2^2 + I_+(z) + (\rho/2) \|A\Delta \theta - b + z + u\|_2^2
\]
The ADMM algorithm

- The scaled ADMM iterations:

\[
\Delta \theta_{k+1} = (J^T J + \lambda I_n + \rho A^T A)^{-1} (J^T \Delta x - \rho A^T (z_k - b + u_k))
\]

\[
z_{k+1} = \max \{0, (-A \Delta \theta_{k+1} + b - u_k)\}
\]

\[
u_{k+1} = u_k + A \Delta \theta_{k+1} - b + z_{k+1}
\]

**Theorem 1:** The constrained redundancy resolution problem with these ADMM iterations converges to zero at **linear rate** for all values of the AL penalty parameter \(\rho\) if:

(a) \(\rho\) is a real positive value \((\rho \in R > 0)\).
(b) damping constant \(\lambda\) is a real positive value \((\lambda \in R > 0)\).

- For any choice of AL penalty parameter \(\rho\) algorithm converges at linear rate.
The ADMM algorithm

**Theorem 2:** Assuming all the conditions of Theorem 1 hold and the constraint matrix $A$ is either full row-rank or invertible then the following optimal AL penalty parameter $\rho^*$ results in the fastest convergence:

$$
\rho^* = \frac{1}{\sqrt{\sigma_{\min}(AQ^{-1}A^T)\sigma_{\max}(AQ^{-1}A^T)}}
$$

Where $\sigma_{\min}$ and $\sigma_{\max}$ are the minimum and maximum eigenvalues of $AQ^{-1}A^T$, respectively.

- Furthermore, when rows of $A$ are linearly dependent, $\rho^*$ can still reduce the convergence time if $\sigma_{\min}$ is replaced by the smallest nonzero eigenvalue of $AQ^{-1}A^T$. 
To apply the proposed method for redundancy resolution of a robotic system, Jacobian of the system as well as the constraints need to be defined.

\[
\text{argmin } \| J(\theta) \Delta \theta - \Delta x \|_2^2 + \lambda \| \Delta \theta \|_2^2
\]

Subject to \( A \Delta \theta \leq b \)

\[
J_{\text{Combined}} = [J_{UR5} \quad J_{CDM}],
\]

\( J_{UR5} \in \mathbb{R}^{6 \times 6} \) and \( J_{CDM} \in \mathbb{R}^{6 \times 2} \)

\[
\dot{x}_{UR5 \text{ base}} = J_{\text{Combined}} \dot{\theta}_{\text{Combined}}
\]

\[
\dot{x}_{CDM \text{ tip}} = J_{\text{Combined}} \dot{\theta}_{\text{Combined}}
\]

Defined Constraints

- The RCM constraint:
  - Ensures movements of the CDM base are confined in a *virtual cylinder* around the screw hole axis.
  - It prevents any collision between the CDM base and the screw hole edges during pivoting around the center of the hole.

\[
\begin{bmatrix} v_1 \\ \vdots \\ v_m \end{bmatrix} \cdot \Delta x_c \leq \begin{bmatrix} \varepsilon_{RCM} + v_1 \cdot u \\ \vdots \\ \varepsilon_{RCM} + v_m \cdot u \end{bmatrix}
\]

\[
\Delta x_c = J_{closest \ point} \cdot \Delta \theta_{UR5};
A_1 \in R^{m \times 7}, \Delta \theta_{UR5} \in R^{6 \times 1}
\]
Defined Constraints

- The joint limits constraint:

\[
\begin{bmatrix}
I_6 & 0 \\
0 & 1 \\
-I_6 & 0 \\
0 & -1
\end{bmatrix}
A_2
\Delta \theta 
\leq
\begin{bmatrix}
\Delta \theta_{UR5_{Upper}} \\
9\text{ mm} - \theta_{CDM} \\
-\Delta \theta_{UR5_{Lower}} \\
\theta_{CDM}
\end{bmatrix}
\begin{bmatrix}
b_2
\end{bmatrix}
\]

where \( \Delta \theta \in R^{7 \times 1}, A_2 \in R^{14 \times 7} \).

- The combined constraint matrix:

\[
\begin{bmatrix}
A_1 & 0 \\
A_2 & A
\end{bmatrix}
\Delta \theta 
\leq
\begin{bmatrix}
b_1 \\
\begin{bmatrix} b_2 \end{bmatrix}
\end{bmatrix}, \Delta \theta \in R^{7 \times 1}.
\]
Algorithm 2: Concurrent UR5-CDM Control

1. Define the desired path, the CDM tip error $\varepsilon_{CDM \text{ tip}}$ and $\varepsilon_{RCM}$.  

2. for $k = 1 : size(desired \text{ path})$ do 
   
   while $\|\Delta x_{\text{desired}}\|_2 \geq \varepsilon_{CDM \text{ tip}}$ do 
   
   Query the UR5 and the CDM current joint values and cable length ($\theta \in \mathbb{R}^{7 \times 1}$) 
   
   Calculate $x_{\text{UR5 base}}^{\text{CDM tip}}$ and its distance to the desired position $x_{\text{desired base}}^{\text{UR5 base}}$ using current joint angle: $\Delta x_{\text{desired}} = x_{\text{CDM tip}}^{\text{UR5 base}} - x_{\text{desired}}^{\text{UR5 base}}$ 
   
   Calculate combined Jacobian, matrix $A$ and vector $b$. 
   
   Compute (11) using (12): $\Delta \theta \leftarrow \text{argmin}_{\Delta \theta} \frac{1}{2} \| J(\theta) \Delta \theta - \Delta x \|_2^2 + \frac{1}{2} \| \Delta \theta \|_2^2$ 
   
   Set $\theta \leftarrow \theta_{old} + \Delta \theta$ 
   
   Set $\theta_{old} \leftarrow \theta$ 
   
   Calculate $\Delta x_{\text{desired}}$ 
   
   end 

3. Set $k \leftarrow k + 1$ 

end
Performance Evaluation Using Simulation

- A complex arbitrary 3D-path including 36 waypoints confined in a $7 \times 7 \times 7$ cm$^3$ cubic space based on a patient’s lesion area.

- The goal of the simulation was to track the mentioned 3D-path and satisfy the RCM and joints limit constraints.

- Effect of the following parameters was studied:
  - the maximum allowable CDM tip error (MTE)
  - and the maximum RCM constraint error (RCME)
  - the damping parameter $\lambda$
  - the ADMM parameter $\rho$
The goal of these simulations is to track a path while satisfying the RCM and joint limits constraints.

\[ \lambda = 2e^{-4}, \quad MTE \leq 0.5 \, mm, \quad \text{and} \quad RCM \leq 0.5 \, mm. \]

The ADMM algorithm accomplished the task in 5.62 seconds (0.15 second/point).
Effect of parameter $\rho$ on the convergence rate

- We proved in Theorem 1, convergence of the ADMM algorithm under the mentioned assumptions is independent of parameter $\rho$.
- However, the convergence rate of the algorithm is directly affected by this parameter.
- we ran the algorithm 50 times with different values of $\rho \in [0.05, 10]$ with $\lambda = 2e^{-4}$, $MTE \leq 0.8 \text{ mm}$, and $RCME = 1 \text{ mm}$. 
Effect of parameter $\rho$ on the convergence rate

As we showed in Theorem 2, for tall matrix $\mathbf{A} \in \mathbb{R}^{30 \times 7}$, $\rho^*$ does not return the optimal AL penalty parameter, however, it still provides a sufficiently close estimation of the optimal parameter and results in a fast convergence rate (37 iterations) compared to the ideal case $\rho^\#$ (28 iterations).
Sensitivity to the MTE and RCME ($\lambda = 2e^{-4}$)

- The MTE defines tracking accuracy: the maximum allowable Euclidean distance between the CDM tip and the desired point.
- The RCME: maximum allowable violation from the RCM constraint or radius of the Virtual cylinder around RCM point.
- Smaller MTE and RCME makes the problem harder.

<table>
<thead>
<tr>
<th>MTE (mm)</th>
<th>RCME (mm)</th>
<th>Run time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.5</td>
<td>21.52</td>
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<tr>
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<td>1</td>
<td>14.37</td>
</tr>
<tr>
<td>1.2</td>
<td>1</td>
<td>4.27</td>
</tr>
</tbody>
</table>

The average overall runtime for all ten ADMM simulations in MATLAB, was 13.3 second and the runtime for each waypoint was about 0.37 second.
Milling Simulated Bone Material with FBG Feedback
Conclusion and Future Work

- We developed a constrained optimization algorithm and virtual fixture method to control the position of a coupled continuum robot and 6 DoF robotic arm inside a confined space.

- We proved a *linear convergence rate* and the *optimum penalty parameter* for obtaining the fastest convergence rate for the proposed redundancy resolution approach with the ADMM algorithm.

- We used an experimental-based CDM Jacobian, future study involves *online adaptive/data-driven approaches* to estimate the CDM deformation behavior.

- Further research will investigate integrating system components and evaluating its overall performance in cadavers with artificial (osteolysis-like) lesions behind the well-fixed acetabular implants.
Thanks for your Attention

Questions??