

Mergers of capacity-constrained firms*

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Abstract

We construct a modified measure of gross upward pricing pressure (*GUPPI*) called *ccGUPPI*, or capacity constrained *GUPPI*. Like other measures of upward price pressure, *ccGUPPI* relies only on information local to pre-merger equilibrium, in contrast to merger simulation which requires assumptions about the curvature of demand and a full set of demand elasticities. It also shows, in a very intuitive terms, how binding capacity constraints alter standard merger effects. We evaluate *ccGUPPI*'s accuracy as a predictor of merger price effects and as a merger screen that flags transactions that would generate a specified threshold price effect. Monte Carlo experiments allow us to scrutinize *ccGUPPI*'s performance over wide range of market conditions. The data from the experiments indicate that *ccGUPPI* outperforms standard *GUPPI* in predicting merger price effects.

1 Introduction

Merging firms regularly argue that mergers involving capacity-constrained firms are unlikely to be anticompetitive, even when there is significant demand substitution between the merging firms' products. The authors have heard such claims in connection with mergers before the FTC in a variety

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of industries. Merging fitness gyms recently made such arguments to the UK competition authority.¹ Penn State Hershey and PinnacleHealth hospital systems argued that capacity constraints mitigated antitrust concerns in response to the FTC’s 2016 effort to block their merger in United States District Court.²

Merging parties typically argue that since constrained firms would lower price and increase quantity but for the constraint, a merger involving capacity-constrained firms is unlikely to result in higher prices. They also argue that a capacity-constrained firm is unable to take on additional customers and thus does not impose a competitive constraint on its merger partner, meaning a merger involving a constrained firm and an unconstrained firm would not reduce competition.

While numerous studies point out that mergers involving capacity-constrained firms indeed may increase price,³ the economics literature lacks tools both to predict which mergers will result in price increase and to predict the magnitude of any price increase. We aim to fill this gap. Our paper constructs a version of gross upward pricing pressure (*GUPPI*) modified to account for capacity constraints, which we call *ccGUPPI*, or capacity-constrained *GUPPI*. Like other measures of upward price pressure, *ccGUPPI* relies only on information that is local to pre-merger equilibrium (price, quantity, margins, and demand elasticities of the merging parties’ products). It can qualitatively predict whether or not a merger will increase prices, and it can quantitatively predict the magnitude of merger price effects.

Specifically, we employ *ccGUPPI* to predict whether both merging firms’ constraints will continue to bind post-merger, and thus eliminate merger price effects. Used in this way, *ccGUPPI* provides a diagnostic as to whether a proposed merger between capacity-constrained firms will likely raise prices, irrespective of the curvature of demand. We further show that *ccGUPPI* can be used to predict the magnitude of merger price effects. We compare *ccGUPPI*’s predictions to actual price increases calculated via merger simulation, using a version of the Monte Carlo experiment of Miller et al. (2016 and 2017) modified so that some firms are capacity-constrained prior to the merger. We find that *ccGUPPI* offers excellent predictions of merger price effects when demand is linear or logit,

¹“According to the parties, the fact that they operate at or close to capacity indicates that they are not providing a significant competitive constraint on each other, as neither of them is seeking to win new customers.” See Competition and Markets Authority (2014), “Anticipated combination of Pure Gym Limited and The Gym Limited,” paragraphs 141 and 142, found via Neurohr (2016).

²“...the combination will alleviate Hershey’s capacity constraints and simultaneously allow both hospitals’ physicians to treat more people,” in “Defendants’ Opposition to Plaintiffs’ Motion For An Injunction Pending Appeal,” May 12, 2016, Document 137. See also “Memorandum Opinion and Order,” May 9, 2016, Document 131, both from Case 1:15-cv-02362-JEJ, US District Court for the Middle District of Pennsylvania, and accessed via pacer.gov on 9/22/2017.

³See Froeb et al. (2003), Higgins et al. (2004), Sandford and Sacher (2016), Neurohr (2016), Oxera (2016), Balan et al (2017), and Chen and Li (2018).

and generally underestimates merger price effects when demand is AIDS or log-linear. For all four demand systems, *ccGUPPI* performs better than the next best alternative predictor, *GUPPI*. We further show that when used as a screen to identify mergers that will generate a specified minimum price increase, *ccGUPPI* has a much lower error rate than *GUPPI* when demand is linear, logit, or AIDS, and a roughly similar error rate when demand is log-linear.

Our paper adds to the somewhat sparse literature on mergers between capacity-constrained firms. Froeb et al. (2003) is the best known paper in this literature. Their paper simulates the effects of a hypothetical merger in a industry producing differentiated goods, subject to differing capacity constraints on the merging and non-merging firms. Based on their simulations, they argue that capacity constraints on merging firms attenuate merger effects more than capacity constraints on non-merging firms amplify them. Froeb et al. (2003) is commonly cited by merging parties alleging that capacity constraints would eliminate or mitigate merger price effects. In particular, in the paper's main example, the merging firms are so tightly capacity-constrained that a merger does not increase price at all.⁴ Higgins et al. (2004) discuss a more general model in the same vein as Froeb et al. (2003), and again demonstrate via simulated results that capacity constraints on merging firms may attenuate merger price effects.

Chen and Li (2018) argue that in a Bertrand-Edgeworth setting with identical firms, firms play a pure strategy if capacity constraints are low enough or high enough and a mixed strategy for the intermediate range. A merger both expands this intermediate range in both directions and shifts the distribution of prices within the mixed equilibrium to the right. Consistent with the Froeb et al. (2003) example, Chen and Li find that a merger has no effect on price outside of this intermediate range of capacity values. However, any industry that falls into the pre-merger intermediate range results in a price increase, as does any merger that causes the industry to shift from a pure to a mixed equilibrium.

Other papers discuss merger price effects when one or both merging firm is constrained in the context of a Cournot model (see Balan et al. (2017), Sacher and Sandford (2016)) or a differentiated Bertrand model (see Balan et al. (2017), Neurohr (2016), Oxera (2016)). All point out that if both merging firms are capacity-constrained pre-merger, positive price effects of the merger result if and only if at least one constraint no longer binds pre-merger. Of particular relevance to our paper is Neurohr (2016), which illustrates how the tightness of pre-merger capacity constraints determines the extent to which the constraints attenuate merger price effects, and discusses a measure of tightness that can be used to modify standard notions of upward pricing pressure when both merging firms are constrained pre-merger, but neither is constrained post-merger.

All of the papers discussed thus far point out that the extent to which a merger of capacity-

⁴See tables 2 and 3 of Froeb et al. (2003).

constrained firms increases price depends on whether or not any capacity constraints continue to bind post-merger. However, none of these papers is able to predict whether or not specific mergers are likely to result in capacity constraints no longer binding, and thus none of these papers offers a general prediction for merger price effects that can be applied by practitioners. Our contribution is to fill this gap. We offer a qualitative prediction for whether or not one or both merging firms will be capacity-constrained post-merger, and then offer a predictor of merger price effects equal to the difference in pricing pressure before and after a merger. We view our paper as being of direct use to antitrust practitioners, who can apply *ccGUPPI* to calculate the pricing pressure resulting from specific mergers.

Finally, the literature on upward pricing pressure traces back to Werden (1996) who demonstrates that one can solve for the marginal cost reduction needed to restore pre-merger price irrespective of the functional form of demand. He derives the marginal cost reduction required to offset the change in marginal revenue created by co-ownership. Farrell and Shapiro (2010) translate Werden’s results into accessible terms and argue that antitrust agencies should test whether a merger will increase or decrease prices based on expected efficiencies. Recent work by Jaffe and Weyl (2013) and Miller et al. (2016 and 2017) examine using upward price pressure to predict post-merger price levels; we extend their work to mergers of capacity-constrained firms.

The next section presents a leading example, which shows how capacity constraints alter merger price effects. Section 3 describes the modeling framework and derives the effect on pricing incentives of a merger of one or more capacity-constrained firms. Section 4 describes how we construct *ccGUPPI*. Section 5 describes the Monte Carlo experiments, including the data analysis that evaluates the performance of *ccGUPPI*. The final section discusses our results and conclusions.

2 Leading example

We first consider a numerical example of duopoly firms merging to monopoly. Specifically, suppose firms 1 and 2 produce differentiated but substitutable products at constant marginal cost 0, competing *a la* Bertrand by simultaneously setting price. Firms face the following demand system:

$$\begin{aligned} q_1 &= 10 - p_1 + \frac{1}{2}p_2 \\ q_2 &= 10 - p_2 + \frac{1}{2}p_1 \end{aligned} \tag{1}$$

Absent capacity constraints, the Nash equilibrium of the Bertrand pricing game in which firm i maximizes $\Pi_i = (p_i - c_i)q_i$ is $(p_i, q_i) = (\frac{20}{3}, \frac{20}{3})$ for $i = 1, 2$. Were the two firms to merge the merged entity jointly chooses p_1 and p_2 to maximize $\Pi_1 + \Pi_2$, and post-merger prices and quantities would be $(p_i, q_i) = (10, 5)$ for $i = 1, 2$. Figure 1(a) plots pre-merger best response functions (in red) and

post-merger first order conditions for the merged firm (in blue). In both cases, solid lines correspond to firm 1, and dashed lines to firm 2. Since the merged firm recaptures some of the lost sales from a price increase, it has an additional incentive to raise prices that did not exist before the merger, and thus the post-merger first order conditions are bowed out relative to the pre-merger best response functions, so that the post-merger equilibrium has higher prices.

Now, suppose that each firm has K_i units of capacity, with marginal cost constant for $q_i \leq K_i$ and prohibitively high for $q_i > K_i$. Figure 1(a) sets $K_1 = K_2 = 8$. We divide each figure into four subsets of the (p_1, p_2) space: where firms 1 and 2 are capacity-constrained, respectively, where both are constrained, and where neither is constrained. In figure 1(a), since both the pre- and post-merger equilibria lie in the region in which neither firm is constrained, the capacity constraints have no effect on either.

The example in figure 1(b) is identical, except that $K_1 = K_2 = 4$. This expands the set of prices for which one or both firms is capacity-constrained. Since each of the firms' unconstrained profit-maximizing prices, both pre- and post-merger, would cause demand to exceed capacity, each firm raises price until its demand just equals its productive capacity. Thus, in figure 1(b), each firm optimally sets a price of 12, and sells quantity 4. Here, the constraints are severe enough that the merger has no price effect; each firm is so constrained pre-merger that the incentive to raise price from the constraint exceeds the incentive to raise price coming from the merger and consequent elimination of competition.

Figure 1(c) considers an example where $K_i = 6, i = 1, 2$. Here each firm's constraint binds before the merger but not after. Pre-merger, we have $p_i = 8, i = 1, 2$ while post-merger we have $p_i = 10, i = 1, 2$. Hence, the capacity constraints attenuate the merger price effect by elevating premerger prices, relative to the case in which firms were not capacity-constrained.

Figure 1(d) considers a case with asymmetric capacity ($K_1 = 8$ and $K_2 = 4.5$), so that exactly one firm is constrained, both before and after the merger. Absent the constraints, the pre-merger Nash equilibrium would be located at the intersection of the red best response curves, or $(\frac{20}{3}, \frac{20}{3})$. Since firm 2 is constrained at this point (but not firm 1), firm 2 will increase its price until $q_2 = 4.5$. Since firm 1's best response to a higher p_2 is itself higher, the Nash equilibrium is located at the intersection of firm 1's pre-merger best response curve and the $q_2 = K_2$ locus, or $(p_1, p_2) = (7.3, 9.1)$, with $(q_1, q_2) = (7.3, 4.5)$.

Following the merger, an unconstrained monopolist would set prices of $(p_1, p_2) = (10, 10)$ and $(q_1, q_2) = (5, 5)$; this is the point at which the two blue lines intersect. However, this point is not feasible, as firm 2 would exceed its capacity constraint of 4.5. Hence, p_2 is set so that $10 - p_2 + \frac{1}{2}p_1 =$

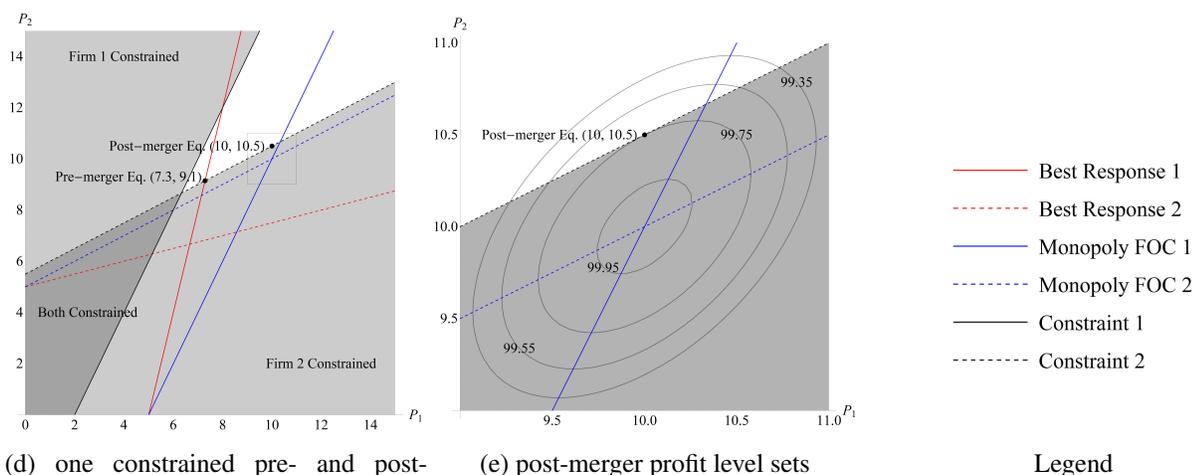
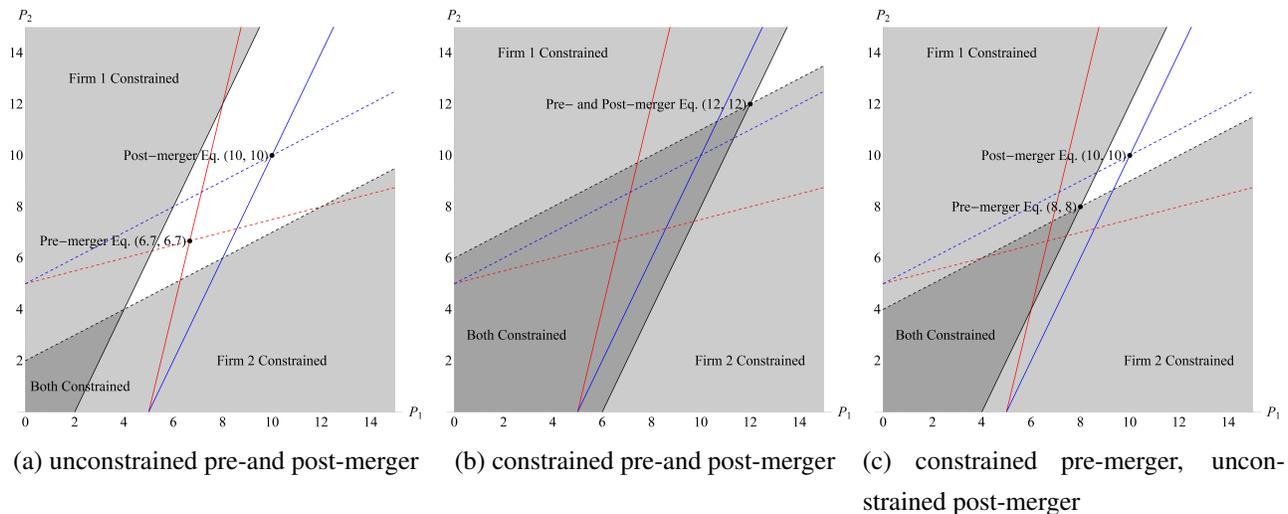


Figure 1: Capacity equals $K_1 = K_2 = 8$ in (a), $K_1 = K_2 = 6$ in (b), and $K_1 = K_2 = 4$ in (c), and $K_1 = 8, K_2 = 4.5$ in (d) and (e). In each case, duopolists under the demand system 1 merge to monopoly. The presence of the capacity constraints do not affect merger price effects in (a), eliminate price effects in (b), and attenuates price effects in (c)-(e).

4.5, while p_1 is the solution to:

$$\begin{aligned} \max_{p_1, p_2} & p_1 * q_1 + p_2 * K_2 & (2) \\ \text{s.t.} & q_1 = 10 - p_1 + \frac{1}{2}p_2 \\ & p_2 = 5.5 + \frac{1}{2}p_1 \end{aligned}$$

In solving (2), the merged firm is choosing the point on the $q_2 = K_2$ locus that maximizes $\pi_1 +$

π_2 . In particular, the monopolist knows an increase in p_1 will lead to an increase in p_2 , since q_2 is increasing in p_1 and decreasing in p_2 . The result is that the merged firm sets prices of $(p_1, p_2) = (10, 10.5)$, meaning that $(q_1, q_2) = (5.25, 4.5)$. Figure 1(e) magnifies the area surrounding the point $(10, 10.5)$ and depicts level sets of the function $\Pi_1 + \Pi_2$, with summed profits increasing in the direction of the point $(10, 10)$. Evidently, the maximum achievable profit on the $q_2 = K_2$ locus is at $(p_1, p_2) = (10, 10.5)$.

We can take away several ideas from this example. First, absent any capacity constraints, a merger of firms 1 and 2 would have led to a 50% price increase, and capacity constraints can attenuate or eliminate the merger price effects depending on how tightly they bind. Second, while capacity constraints generally attenuate merger price effects by elevating pre-merger prices, price still increases following the merger, so long as at least one product is unconstrained post-merger. Indeed, even in figure 1(d)-(e), both p_1 and p_2 increase despite firm 2's constraint binding both before and after the merger. The optimization problem of the merged firm changes when one product is capacity constrained and the other is not, but the merged firm still internalizes increased demand for product 2 following an increase in p_1 , and this increased demand allows for a higher p_2 .

The next section specifies a general model of differentiated Bertrand competition with capacity constraints. We derive the pre- and post-merger equilibrium conditions and then subsequently use those conditions to construct *ccGUPPI*.

3 Model

We study a standard model of price competition among N firms selling differentiated products. Given a vector of prices \mathbf{p} , firm i 's demand is $q_i^D(\mathbf{p})$, while it's total cost function to produce quantity q is $c_i(q)$. Thus, firm i 's profit is given by $\pi_i = q_i^D(\mathbf{p})p_i - c_i(q_i^D(\mathbf{p}))$. We assume that the demand function q_i^D is differentiable, decreasing in p_i , increasing in p_j for $j \neq i$, and satisfying $\frac{\partial q_i^D}{\partial p_i} + p_i \frac{\partial^2 q_i^D}{\partial p_i^2} \leq 0$ for all $p_i > 0$, so that each firm's demand becomes more elastic as price increases.

Each firm has access to a constant marginal cost production technology capable of producing K_i units (e.g., a factory). We refer to K_i as a firm's *capacity*. Each firm additionally has access to a higher cost production technology of unlimited capacity (e.g., buying or importing the product instead of producing it, or repurposing a factory producing a different good). We refer to this additional production technology as a firm's *flex capacity*. We assume that a firm's marginal cost increases by

$\gamma > 1$ once it begins using its flex capacity.⁵ Thus, equation (3) describes a firm's total cost curve:

$$c_i(q) = \begin{cases} c_i q & \text{if } q \leq K_i \\ c_i K_i + \gamma_i c_i (q - K_i) & \text{if } q > K_i \end{cases} \quad (3)$$

We make two simplifying assumptions, one to guarantee the existence of a pure-strategy Nash equilibrium in prices, and one to simplify discussion of capacity.

Assumption 1: For any price vector \mathbf{p} , a firm's quantity sold is $q_i^D(\mathbf{p})$

Assumption 2: $\gamma_i > p_i^m$, where p_i^m denotes i 's monopoly price

Assumption 1 dictates that once prices \mathbf{p} are set, a firm's demand $q_i^D(\mathbf{p})$ determines its quantity sold. The alternative, allowing firms to re-optimize over quantity once all prices are set, leads to non-existence of pure strategy Nash equilibria, and mixed equilibria that depend on an assumed rationing rule.⁶ By Dastidar (1997), this assumption is justified when prices are set by sealed bid tenders, or when there are large costs to turning away customers. Assumption 1 obviates the need for a rationing rule and ensures the existence of a pure-strategy Nash equilibrium, with each firm producing the maximum quantity such that marginal revenue is greater than or equal to marginal cost (holding with equality for unconstrained firms). Assumption 1 is ubiquitous in the literature on oligopolies, and appears in models with and without capacity constraints.⁷ Assumption 2 ensures that no firm will use its flex capacity in equilibrium. While this assumption is not necessary, it simplifies language: in equilibrium, a firm is either *capacity-constrained* ($q_i = K_i$) or *unconstrained* ($q_i < K_i$).⁸

We proceed by solving the model both before and after a merger of firms 1 and 2. Then, we study how the change in incentives generated by the merger vary in whether or not each merging firm is capacity-constrained prior to the merger.

⁵Dixit (1980) is the earliest example we know of to include a stepped cost function to model capacity constraints. See also Maggi (1996) and Boccard and Wauthy (2000), each of which uses the same cost function we do.

⁶Suppose there were a pure strategy equilibrium under this alternative assumption. Then, each unconstrained firm would set price so that demand equals both marginal revenue and marginal cost, while constrained firms would price so that demand equals capacity. But then any one unconstrained firm would have an incentive to increase price slightly, causing all other firms to choose a quantity less than demand. Some of this quantity would then be reallocated towards the firm who increased price, according to the assumed rationing rule. As a marginal price increase would have no direct effect on an optimizing firm's profit, the total effect on profit of the price increase plus the additional quantity must be positive. Thus, there can be no pure strategy equilibrium under the alternative assumption. See Shapley and Shubik (1969) for a fuller discussion of potential non-existence of equilibrium.

⁷In addition to Dastidar (1997), see Bulow et al. (1985), Vives (1990), Dixon (1990), Dastidar (1995), and Chen (2009).

⁸A firm with $q_i > K_i$ (absent assumption 2) could also be said to be unconstrained with marginal cost $\gamma_i c_i$. However, a merger involving this firm could lower its equilibrium quantity to be less than or equal to K_i . It is this nuisance case we avoid with assumption 2.

3.1 Pre-merger equilibrium

When all N firms are separately owned, each firm i takes all other prices p_{-i} as given, and chooses p_i to maximize profits. Under assumption 1, firm i 's profits are given by $q_i^D(\mathbf{p})p_i - c_i(q_i^D(\mathbf{p}))$. Let $q_i^{-1}(K_i, \mathbf{p}_{-i})$ denote the price p_i at which $q_i^D = K_i$, and below which $q_i^D > K_i$, given prices \mathbf{p}_{-i} . Under assumption 2, all firms set price $p_i \geq q_i^{-1}(K_i, \mathbf{p}_{-i})$ and each firm has constant marginal cost of c_i . Thus, firm i 's pre-merger maximization problem is described by:

$$\begin{aligned} \max_{p_i} q_i^D(\mathbf{p})(p_i - c_i) \\ \text{s.t. } p_i \geq q_i^{-1}(K_i, \mathbf{p}_{-i}) \end{aligned} \quad (4)$$

For an unconstrained firm i , the first-order condition for (4) is:

$$\begin{aligned} \frac{\partial \pi_i}{\partial p_i} = q_i(p) + \frac{\partial q_i}{\partial p_i}(p_i - c_i) = 0 \\ \Rightarrow \frac{p_i - c_i}{p_i} = -\frac{1}{\epsilon_{ii}} \end{aligned} \quad (5)$$

where $\epsilon_{ii} = \frac{\partial q_i}{\partial p_i} \frac{p_i}{q_i}$ denotes firm i 's own-price elasticity. Equation (5), relating firm i 's margin over cost to its elasticity of demand, is the well-known Lerner condition.

Now, suppose that firm i is capacity-constrained in equilibrium, so that the constraint in (4) binds with equality. Thus, from (5), $\frac{p_i - c_i}{p_i} > -\frac{1}{\epsilon_{ii}}$, so the Lerner condition no longer holds. Let $\lambda_i = \frac{p_i - c_i}{p_i} + \frac{1}{\epsilon_{ii}} > 0$ be the difference between the left- and right-hand sides of (5), or the amount by which the Lerner condition is violated. Then, for any constrained firm, we have:

$$\frac{p_i - c_i}{p_i} = -\frac{1}{\epsilon_{ii}} + \lambda_i \quad (6)$$

The quantity λ_i is a measure of upward pricing pressure due to the capacity constraint K_i binding. In the following section, we demonstrate that when a constrained firm merges with another firm, λ_i is less than than upward pricing pressure from the merger (derived below) if and only if the merger causes a firm that was capacity-constrained pre-merger to raise its price to the point where it is no longer constrained post-merger.

A Nash equilibrium is a price vector \mathbf{p} such that the first order condition (5) holds for all firms for which $q_i^D(\mathbf{p}) < K_i$, and such that no firm's quantity demand exceeds its capacity (i.e. $q_i^D(\mathbf{p}) \leq K_i$ for all i).

3.2 Post-merger equilibrium

We now consider a merger of firms 1 and 2, and derive post-merger analogues of pricing equations (5) and (6). Under assumption 2, the merged firm jointly chooses prices for products 1 and 2 to

maximize $q_1(\mathbf{p})(p_1 - c_1) + q_2(\mathbf{p})(p_2 - c_2)$, subject to the constraint that neither product exceed its capacity. We solve for pricing equations when the merging firm is capacity-constrained in choosing zero, one, or both of its prices. The next section develops a prediction for which products will be capacity-constrained post-merger, as a function of pre-merger information.

First, suppose that neither product is capacity-constrained post-merger. Then, the merged firm's profits are maximized for prices p_1 and p_2 satisfying the first order conditions below:

$$\begin{aligned} \frac{\partial \pi_i}{\partial p_i} &= q_i(p) + \frac{\partial q_i}{\partial p_i}(p_i - c_i) + \frac{\partial q_j}{\partial p_i}(p_j - c_j) = 0 \\ \Rightarrow \frac{p_i - c_i}{p_i} &= - \left(\frac{1}{\epsilon_{ii}} + \frac{\frac{\partial q_j}{\partial p_i} p_j}{\frac{\partial q_i}{\partial p_i} p_i} \frac{p_j - c_j}{p_j} \right) \text{ for } i = 1, 2 \end{aligned} \quad (7)$$

Let $D_{ij} = -\frac{\partial q_j}{\partial p_i} / \frac{\partial q_i}{\partial p_i}$ denote the diversion ratio between firms i and j , or the fraction of firm i 's marginal customers who view firm j as their next-best option. Then, let $GUPPI_i = D_{ij} \frac{p_j}{p_i} \frac{p_j - c_j}{p_j}$. $GUPPI_i$, well known in the literature on antitrust economics and commonly used by antitrust practitioners,⁹ is a measure of upward pricing pressure due to a merger. Its terms are intuitive: following a merger, firm i has an incentive to increase price because some of the customers it loses will be recaptured by its former rival, and the value of these customers depends on relative prices and the former rival's margin. We rewrite equation (7) below. If neither of the merged firm's capacity constraints are binding, it sets prices p_1 and p_2 according to:

$$\frac{p_i - c_i}{p_i} = -\frac{1}{\epsilon_{ii}} + GUPPI_i \text{ for } i = 1, 2 \quad (8)$$

Next, we consider what happens if the merged firm is capacity-constrained in exactly one of the formerly separate products. Without loss of generality, assume product 2 is capacity-constrained following a merger, while product 1 is unconstrained. In this case, the merged firm sets $p_2 = q_2^{-1}(K_2, \mathbf{p})$, meaning that the merger does not directly alter pricing incentives for product 2. The quantity λ_2 from equation (6) reflects firm 2's upward pricing pressure following the merger. Thus, if a firm is constrained both before and after a merger, its upward pricing pressure is equal to λ_i in both cases.

That firm 2 prices so as to set $q_2^D = K_2$ following the merger has been incorrectly interpreted by merging parties to mean that firm 1's incentives are unaffected by the merger, as well. As when product 2 is unconstrained, an increase in p_1 diverts some of product 1's customers to product 2.

⁹For example, "As a general matter, Dollar Tree and Family Dollar stores with relatively low GUPPIs suggested that the transaction was unlikely to harm competition... Conversely, Dollar Tree and Family Dollar stores with relatively high GUPPIs suggested that the transaction was likely to harm competition," from "Statement of the Federal Trade Commission In the Matter of Dollar Tree, Inc. and Family Dollar Stores, Inc.," July 13, 2015, accessed on October 2, 2017 from www.ftc.gov/public-statements/2015/07/statement-federal-trade-commission-matter-dollar-tree-inc-family-dollar.

However, since firm 2 is capacity-constrained, the merged firm is unable to capture these diverted customers in the form of a greater quantity q_2 . Instead, customers diverted to product 2 bid up the price at which product 2 is exactly at capacity, enabling the merged firm to charge a higher price for product 2 to sell the same quantity. Some of product 2's marginal customers will divert to product 1 in response to this price increase, further increasing the merged firm's profits. Alternatively, if q_2 is fixed at K_2 , the merged firm sets p_1 so that the value of a reduction in q_1 equals the value of an increase in $p_2 = q_2^{-1}(K_2, \mathbf{p}_{-2})$ caused by increasing p_1 .

This effect — a merged firm increasing price when exactly one of its two products is constrained — generates a new first-order condition, distinct from (8). In this case, the merged firm sets $p_j = q_j^{-1}(K_j, \mathbf{p}_{-j})$ for the constrained product and sets price for its remaining product to satisfy:

$$\max_{p_i} \pi_i + \pi_j = q_i^D(\mathbf{p})(p_i - c_i) + K_j(p_j - c_j) \text{ s.t. } p_j = q_j^{-1}(K_j, \mathbf{p}_{-j})$$

The first-order condition for maximizing $\pi_i + \pi_j$ with respect to p_i is then:

$$\frac{\partial(\pi_i + \pi_j)}{\partial p_i} = q_i^D(\mathbf{p}) + \left(\frac{\partial q_i^D}{\partial p_i} + \frac{\partial q_i^D}{\partial p_j} \frac{\partial p_j}{\partial p_i} \right) (p_i - c_i) + K_j \frac{\partial p_j}{\partial p_i} = 0 \quad (9)$$

where $\frac{\partial p_j}{\partial p_i}$ reflects how p_j responds to a small change in p_i along the locus of points satisfying $p_j = q_j^{-1}(K_j, \mathbf{p}_{-j})$. Applying the implicit function theorem, $\frac{\partial p_j}{\partial p_i} = -\frac{\frac{\partial q_j^D}{\partial p_i}}{\frac{\partial q_j^D}{\partial p_j}}$. We can then rewrite (9) as:

$$\begin{aligned} \frac{\partial(\pi_i + \pi_j)}{\partial p_i} &= q_i^D + \left(\frac{\partial q_i^D}{\partial p_i} - \frac{\partial q_i^D}{\partial p_j} \frac{\frac{\partial q_j^D}{\partial p_i}}{\frac{\partial q_j^D}{\partial p_j}} \right) (p_i - c_i) - K_j \frac{\frac{\partial q_j^D}{\partial p_i}}{\frac{\partial q_j^D}{\partial p_j}} = 0 \\ \iff \frac{p_i - c_i}{p_i} &= -\frac{1}{\epsilon_{ii}} + \theta_i, \text{ where } \theta_i = m_i D_{ij} D_{ji} - \frac{p_j}{p_i} D_{ij} \frac{1}{\epsilon_{jj}} \end{aligned} \quad (10)$$

The term θ_i defines a second source of upward pricing pressure, describing the change in incentive for a firm which is not capacity-constrained post-merger, but whose former rival is. θ_i consists of two terms, both positive (as $\epsilon_{jj} < 0$), meaning that both increase i 's margin relative to its pre-merger first order condition (5). The first term captures the value of customers diverted from j to i following an increase in p_i and a consequent increase in p_j . The second term captures the value of the increase in p_j caused by the increase in p_i , holding fixed j 's quantity at K_j . Note that the second term is smaller the more elastic j 's demand is, reflecting the fact that a smaller increase in p_j would be needed to sell out capacity the more elastic its demand is. The second term is also increasing in diversion from i to j and in the relative price of j .

Finally, if both of the merged firm's products are capacity constrained, it simply sets price for each such that quantity demanded equals capacity, or $p_i = q_i^{-1}(K_i, \mathbf{p}_{-1})$ for $i = 1, 2$. In this case,

equation (6) holds, so that each firm has post-merger upward pricing pressure equal to λ_i . It is direct that a merged firm can be capacity-constrained in both products post-merger only if both firms were capacity-constrained pre-merger. In this case, the merger does not increase the upward pricing pressure of either product.

Following a merger of firms 1 and 2, a price vector \mathbf{p} comprises a Nash equilibrium if and only if:

- (i)— First order condition (8) is satisfied if $q_i^D(\mathbf{p}) < K_i$ for $i = 1, 2$
- (ii)— First order condition (10) is satisfied for i if $q_j^D(\mathbf{p}) = K_j$, $i, j = 1, 2, j \neq i$
- (iii)— First order condition (5) is satisfied for any non-merging firm i if $q_i^D(\mathbf{p}) < K_i$
- (iv)— No firm i has $q_i^D(\mathbf{p}) > K_i$

The next section develops a qualitative prediction about whether or not capacity constraints that bind pre-merger will continue to bind post-merger, based on whether or not post-merger pricing pressure exceeds pre-merger pricing pressure. Then, we add a quantitative measure of pricing pressure from the merger equal to the difference in pre- and post-merger pricing pressure. The result, *ccGUPPI*, modifies conventional *GUPPI* to analyze the price effect of mergers between one or more capacity-constrained firms.

4 Upward pricing pressure with capacity constraints

The previous section identifies pricing pressure from a merger of firms 1 and 2 as equal to *GUPPI* or θ if a firm is unconstrained post-merger, the former applying if its rival is also unconstrained, the latter if its rival is constrained post-merger. If a firm is capacity-constrained either pre- or post-merger, its upward pricing pressure from the constraint equals λ . Applying pricing pressure analysis to a merger of constrained firms thus depends on being able to predict which firms will be capacity-constrained post-merger. We now develop such a qualitative prediction.

We assume that a merger causes each merging firm's optimal quantity to decrease. Then, it is direct that a firm may be capacity-constrained following a merger only if it was similarly constrained prior to the merger. The problem of determining which merging firms are capacity-constrained post-merger then reduces to determining which merging firms that were constrained pre-merger remain constrained post-merger.¹⁰

¹⁰It is not clear that this assumption is satisfied generally across demand systems satisfying the assumptions at the beginning of section 3. See, for example, McElroy (1991), page 75: "...the presence of cross effects on the demand side leads (in general) to indeterminacy of quantity effects and rules out general statements even if the merger causes all prices to rise." Should our assumption not hold, it is possible that a firm that is unconstrained prior to the merger could

First, we show in proposition 1 that if both firms 1 and 2 are capacity-constrained pre-merger, they will both be constrained post merger if and only if one of the following two conditions are met: (i)- $\theta_1 \leq \lambda_1$ and $GUPPI_2 \leq \lambda_2$, (ii)- $\theta_1 \leq \lambda_1$ and $GUPPI_2 \leq \lambda_2$. As a merger does not directly affect the incentives of non-merging firms, it follows immediately that a merger of two capacity-constrained firms increases price if and only if condition (i) or (ii) is satisfied. Then, we argue that if both conditions fail, or if one firm was unconstrained pre-merger, whether a merging firm continues to be constrained post-merger can be approximated by comparing the size of post-merger pricing pressure ($GUPPI$ or θ , as applicable) to the size of pre-merger pricing pressure (λ). Finally, we define a firm's $ccGUPPI$ as the difference between its post-merger and pre-merger pricing pressure.

We now state and prove proposition 1, which identifies necessary and sufficient conditions for a merger of two capacity-constrained firms to result in a price increase. The proposition works by identifying when a binding pre-merger capacity constraint will cease to bind following a merger.

Proposition 1. *Suppose that, given equilibrium prices \mathbf{p}^* , firms 1 and 2 are capacity-constrained, so that $q_i^D(\mathbf{p}^*) = K_i$, $i = 1, 2$. Following a merger of firms 1 and 2, all firms set the same price \mathbf{p}^* if and only if at least one of the following two conditions hold:*

$$(i): \theta_1 \leq \lambda_1 \text{ and } GUPPI_2 \leq \lambda_2$$

$$(ii): \theta_2 \leq \lambda_2 \text{ and } GUPPI_1 \leq \lambda_1$$

Proof If: Suppose that (i) holds. Since both firms are constrained prior to the merger, $\frac{p_i^* - c_i}{p_i^*} = -\frac{1}{\epsilon_{ii}} + \lambda_i$ for $i = 1, 2$.

Supposing (for now) that firm 2 is capacity-constrained following the merger, firm 1 would set price via (10) were it unconstrained following the merger. Since $\frac{p_1 - c_1}{p_1}$ is increasing in p_1 and $-\frac{1}{\epsilon_{11}}$ decreasing in p_1 , and since $\lambda_1 \leq \theta_1$, the price that satisfies (8) is less than p_1^* , firm 1's pre-merger price, for any constant \mathbf{p}_{-1} . Firm 1 would more than sell out its capacity at this lower price. Thus, unless another firm changes its price following the merger, firm 1 would continue to set price p_1^* following the merger.

Were firm 2 to be unconstrained post-merger, it would set price via either (8) or (10). By the same argument as in the previous paragraph, and price satisfying either equation would cause firm 2 to more than sell out its capacity, implying that unless another firm changes its price following the merger, firm 2 would continue to set price p_2^* . An identical proof applies if (ii) holds.

Only if: Suppose, that both firms are constrained following a merger but that neither (i) nor (ii) is true. But then either firm can set a post-merger price different than p_i^* satisfying (10), at which increase its quantity and become constrained following the merger. However, such cases appear to be of minimal practical importance in our simulated data, both because they do not occur often and because when they do occur, $ccGUPPI$ as defined in definition 2 appears to adequately estimate price effects.

firm i is unconstrained. Given the assumptions on demand, such a price maximizes firm i 's profit, contradicting the premise that firm i is constrained following the merger. ■

Proposition 1 allows a precise prediction of whether a merger of two capacity-constrained firms will result in a price increase. For mergers involving two capacity-constrained firms that fail the condition of proposition 1, or for mergers involving exactly one capacity-constrained firm, we implement a similar scheme to determine which capacity constraints that bind pre-merger continue to bind post-merger.

First, suppose that both firms are capacity-constrained pre-merger, but neither condition (i) nor (ii) of proposition 1 hold. If $\theta_i > \lambda_i$, $i \in \{1, 2\}$ then firm i will no longer be constrained post-merger. If, additionally, $GUPPI_j > \lambda_j$, $j \neq i$, then the merger will also cause the merged firm to increase p_j by enough that product j is no longer constrained post-merger.¹¹

Next, suppose that exactly one firm i is constrained pre-merger. Firm i 's pricing pressure from the merger equals $GUPPI_i$, while its pre-merger price is elevated by λ_i due to its pre-merger capacity constraint. Thus, i becomes unconstrained post-merger if $GUPPI_i > \lambda_i$.

Unlike the conditions in proposition 1, those in the previous two paragraphs only approximately characterize the status of post-merger capacity constraints. Feedback effects, in which rivals' actions affect each others' incentives, account for the non-exactness of the above conditions. For example, suppose that exactly one firm, firm i , is constrained pre-merger. The reason that the inequality $GUPPI_i > \lambda_i$ does not exactly characterize i 's post-merger capacity constraint is that firm j has an incentive to increase its price regardless of whether or not j 's constraint continues to bind; this increase in p_j in turn affects i 's pricing incentive, in a way not reflected by whether or not $GUPPI_i$ is greater than λ_i , both evaluated at pre-merger prices. These spillover effects are absent under the conditions in proposition 1, under which neither firm increases price post-merger.

With the qualitative predictions described above, we are ready to define $ccGUPPI$, the upward pricing pressure resulting from a merger of one or more capacity-constrained firms. $ccGUPPI$ has the same intuitive construction as other measures of pricing pressure (e.g. Farrell and Shapiro (2010)). Specifically, $ccGUPPI$ equals the change in pricing incentive from before to after the merger. We implement this by differencing the conditions (5), (6), (8), and (10), evaluated at premerger prices, or:

$$ccGUPPI_i = \left(\frac{p_1 - c_1}{p_1} + \frac{1}{\epsilon_{11}} \right) \Big|_{p_1^{pre}}^{post} - \left(\frac{p_1 - c_1}{p_1} + \frac{1}{\epsilon_{11}} \right) \Big|_{p_1^{pre}}^{pre} \quad (11)$$

¹¹In evaluating each of these inequalities, recall that a capacity-constrained firm sets price so that its demand is exactly equal to its capacity. Hence, any further increase in price will cause that firm to become unconstrained.

The first term of $ccGUPPI_i$ equals $GUPPI_i$, θ_i , or λ_i depending on whether neither, one, or both merging firms are unconstrained post-merger. The second term equals λ_i or 0 depending on whether that firm is constrained pre-merger or not. We apply the qualitative predictions discussed above and in proposition 1 to determine the form of the first term, which the form of the second term directly reflects observable information.

Definition 2 explicitly defines $ccGUPPI$ in terms of $GUPPI$, θ , λ and pre-merger constraints. In so defining $ccGUPPI$ we note that there are nine possible cases (e.g., both firms constrained before, neither constrained after), which we separate by row. We number the cases arbitrarily, and further name them by a 2×2 matrix, whose element $(i, 1)$ equals 1 if firm i is constrained prior to the merger and 0 otherwise, and whose element $(i, 2)$ equals 1 if firm i is constrained post-merger and 0 otherwise, $i = 1, 2$.

Definition 2. $ccGUPPI$ is defined as follows. The first two columns describe our qualitative prediction of which pre-merger capacity constraints continue to bind post-merger, while the third and fourth column describe $ccGUPPI_1$ and $ccGUPPI_2$, respectively.

Criteria	Case	$ccGUPPI_1$	$ccGUPPI_2$
$q_1 = K_1, q_2 < K_2$ $GUPPI_1 \leq \lambda_1$	1 $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$	0	θ
$q_1 = K_1, q_2 < K_2$ $GUPPI_1 > \lambda_1$	2 $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	$GUPPI_1 - \lambda_1$	$GUPPI_2$
$q_1 = K_1, q_2 = K_2$ $GUPPI_1 \leq \lambda_1, \theta_2 > \lambda_2$	3 $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$	0	$\theta_2 - \lambda_2$
$q_1 = K_1, q_2 = K_2$ $GUPPI_1 > \lambda_1, GUPPI_2 > \lambda_2$	4 $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$	$GUPPI_1 - \lambda_1$	$GUPPI_2 - \lambda_2$
$q_1 < K_1, q_2 = K_2$ $GUPPI_2 > \lambda_2$	5 $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$	$GUPPI_1$	$GUPPI_2 - \lambda_2$
$q_1 = K_1, q_2 = K_2$ $\theta_1 \leq \lambda_1$ and $GUPPI_2 \leq \lambda_2$ or $\theta_2 \leq \lambda_2$ and $GUPPI_1 \leq \lambda_1$	6 $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	0	0
$q_1 = K_1, q_2 = K_2$ $\theta_1 > \lambda_1, GUPPI_2 \leq \lambda_2$	7 $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$	$\theta_1 - \lambda_1$	0
$q_1 < K_1, q_2 = K_2$ $GUPPI_2 \leq \lambda_2$	8 $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$	θ_1	0
$q_1 < K_1, q_2 < K_2$	9 $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$GUPPI_1$	$GUPPI_2$

where $\lambda_i = \frac{p_i - c_i}{p_i} + \frac{1}{\epsilon_{ii}}$, $GUPPI_i = D_{ij} \frac{p_j}{p_i} \frac{p_j - c_j}{p_j}$, and $\theta_i = m_i D_{ij} D_{ji} - \frac{p_j}{p_i} D_{ij} \frac{1}{\epsilon_{jj}}$.

Like other measures of upward pricing pressure, $ccGUPPI$ can be compared to the magnitude of any expected cost-saving efficiencies that would result from the merger to determine if the merger's net upward pricing pressure is positive or negative (see Werden (1996) for a discussion of comparing upward pricing pressure to efficiencies). We say that a merger of one or more capacity-constrained firms has net upward pricing pressure if $ccGUPPI_i > \frac{\Delta c_i}{p_i}$ for merging firm i . Such a merger will generate a price increase, regardless of the particulars of how cost increases are passed through to consumers.

Returning to the example considered in section 2, a symmetric duopoly in which $q_i = 10 - p_i + \frac{1}{2}p_j$, and $c_1 = c_2 = 0$, figure 2 plots the nine cases as regions of the (K_1, K_2) parameter space, first using merger simulation in subfigure 2a (with full knowledge of the demand system and thus post-merger equilibrium) and then using the criteria in definition 2 under $ccGUPPI$. The two pictures are identical, meaning that $ccGUPPI$ perfectly identifies the nine regions in this one example.¹²

In the next section we describe Monte Carlo experiments that allow us to compare $ccGUPPI$ to both standard $GUPPI$ and to simulated merger effects under four different demand systems. The experiments allow us to do so over a large space of randomly drawn market characteristics.

5 Applying $ccGUPPI$ to predict merger price effects

Measures of upward pricing pressure, including $ccGUPPI$, measure the change in incentives (specifically, the change in the first order condition) due to internalizing the effect of a firm's own price on its former rival's profits. To translate this change in incentives into a price increase, Jaffe and Weyl (2013) suggest multiplying the vector of pricing pressure terms by a pass-through matrix equal to the negative inverse Jacobian of the first order conditions, and show that the difference between this first order approximation and the actual price increase is quite small. However, the Jaffe and Weyl (2013) pass through matrix is often difficult to implement in applied antitrust settings.

Miller et al. (2017) suggest replacing the Jaffe and Weyl pass-through matrix with the identity matrix, and use simulated data to show that $GUPPI$ multiplied by the identity matrix is a reasonably good predictor of merger price effects. Following Miller et al. (2017) both in setting pass-through equal to the identity matrix, and in repurposing their simulated data generating process, we show that $ccGUPPI_i$ is an excellent predictor of firm i 's merger price effects when one or both merging

¹²Perfect prediction of the regions appears to be an artifact of symmetric linear demand, although for more general demand systems the borders that are only approximated appear to be quite accurate. A future version will measure the accuracy with which $ccGUPPI$ predicts case in our simulated data.

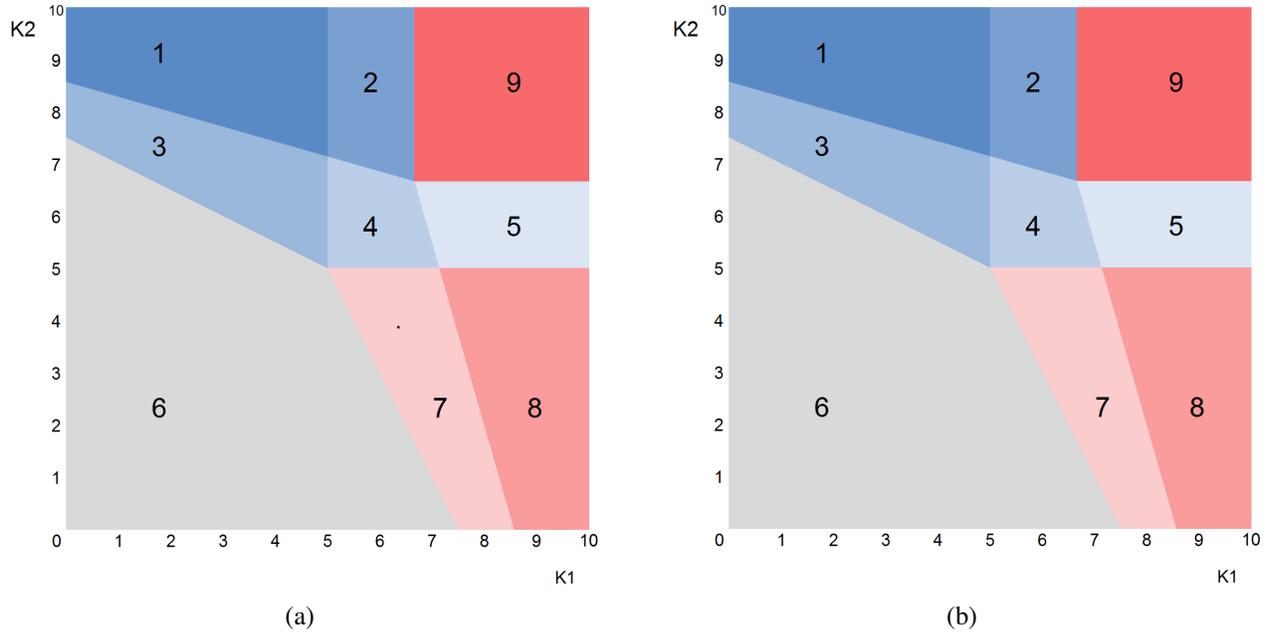


Figure 2: Panel (a) depicts cases 1-9 as determined by merger simulation, with full knowledge of the demand system. Panel (b) depicts regions 1-9 as determined by definition 2, which uses only pre-merger information, and no knowledge of the demand system. The underlying demand system is $q_i = 10 - p_i + .5 * p_j$ for $i = 1, 2$, with $c_i = 0$.

firms is constrained if demand is linear or logit. If demand is more convex (i.e. AIDS or log-linear), *ccGUPPI* is a good predictor, and clearly better than ignoring the capacity constraints and using standard *GUPPI*.

5.1 Simulated data

We construct a dataset where each observation, or random draw of data, represents a market consisting of six firms, and assumes that firms 1 and 2 will merge. We calibrate four different demand systems (linear, logit, AIDS, and log-linear) with each draw of data. While the demand systems differ in functional form, firms have the same pre-merger prices, quantities, margins, and demand elasticities under each system. We make 10,000 draws allowing us to examine *ccGUPPI*'s performance as a predictor of merger price effects relative to 40,000 potential mergers, each defined by a draw of data and a specific demand system.

Our data generating processes directly follows Miller et al. (2016 and 2017). First, we randomly draw market shares for six firms and an outside good. We also draw the margin of a single firm. Then we calibrate the parameters of a logit demand system based on market shares, the margin of the single

firm, and prices normalized to unity. Next, we calibrate linear, AIDS, and log-linear demand systems using the market shares, prices, and demand slopes from the logit calibration.

The aforementioned steps mirror those taken of Miller et al. (2016 and 2017), and would be sufficient to simulate a merger under each of the four demand systems. We augment the data generating process by randomly assigning each firm to be either capacity-constrained or not, and separately draw each constrained firm’s margin. Each unconstrained firm has marginal costs implied by marginal revenue (just as it would in a model without capacity constraints) and each constrained firm has marginal cost that are greater than zero and less than marginal revenue.

Finally, we use each calibrated demand systems and marginal cost vector to compute the optimal post-merger price vector, and compare the simulated price increases to both *ccGUPPI* and conventional *GUPPI*.¹³ Upward price pressure depends only on information that is local to pre-merger equilibrium (prices, quantities, marginal costs, and demand slopes), thus there is one *ccGUPPI* and *GUPPI* per random draw of data. We repeat this exercise 10,000 times.

The specific steps of the data generating process are as follows:

1. Draw seven random numbers from a uniform distribution between zero and one. Compute market shares for six firms and an outside good as a percentage of the sum of the seven random draws.
2. Draw a single firm’s margin from a uniform distribution between 0.2 and 0.8. Apply this margin to firm 1.
3. Calibrate the parameters of a logit demand system based on the single margin, market shares, and prices normalized to unity.
4. Calibrate linear, AIDS, and log linear demand systems using the prices, quantities, and elasticities from the logit system. Prices, quantities, and a full set of elasticities identify the parameters of the additional three systems.
5. Randomly assign capacity constraints based on six 0/1 draws from a Bernoulli distribution.
6. Marginal revenue implies the marginal cost of each unconstrained firm. The product of marginal revenue and an additional random draw from a uniform distribution between 0.2 and one sets the marginal cost of each constrained firm.

¹³When we refer to “conventional *GUPPI*,” we mean $GUPPI_i = D_{ij} \frac{p_j}{p_i} \frac{p_j - c_j}{p_j}$, and ignoring the fact that capacity-constraints may elevate p_j or p_i . An antitrust practitioner may calculate such a measure of pricing pressure if it is unclear whether or not a firm is capacity-constrained, or if he/she believes the capacity constraints to have minimal effect on equilibrium prices.

7. Calculate $GUPPI$ and $ccGUPPI$ for a merger of the first two firms in each market.
8. Analytically calculate the post-merger price vector under each demand system.
9. Repeat steps (1) through (8) 10,000 times.

The resulting 40,000 mergers, each defined by a randomly drawn pre-merger equilibrium and specific demand system, allow us to assess the accuracy of $ccGUPPI$ across the four different demand systems and an array of potential market conditions.

5.2 Summary statistics

Table 1 summarizes the simulated data. It reports order statistics for firm 1's market share, margin, and demand elasticity. The distributions of these variables for the other firms are essentially the same, because the data generating process is the same for all firms. The median market share, 14.2 percent, reflects that there are six firms and an outside good. The median margin is 56 percent and the median elasticity is 2.3. Note that the traditional Lerner index relationship between margins and elasticity does not always hold because capacity-constrained firms have marginal cost below marginal revenue. The median diversion ratio from firm 1 to 2 is 0.16.

Table 1: Order Statistics

	p50	p10	p25	p75	p90
Market Share	0.142	0.035	0.079	0.195	0.240
Own-Price Elasticity	2.271	1.491	1.752	3.114	4.018
Diversion Ratio	0.163	0.041	0.092	0.224	0.273
Margin	0.563	0.291	0.405	0.714	0.811
$ccGUPPI$	0.024	0.000	0.000	0.073	0.117
$GUPPI$	0.075	0.017	0.039	0.123	0.165
Logit Price Effect	0.027	0.000	0.004	0.068	0.109
Linear Price Effect	0.021	0.000	0.003	0.050	0.083
Log-Lin Price Effect	0.081	0.000	0.011	0.246	0.630
AIDS Price Effect	0.048	0.000	0.006	0.147	0.382

Table 1 also shows the upward price pressure and simulated price effect for firm 1. The median $GUPPI$ is 7.5 percent, while the median $ccGUPPI$ is only 2.4 percent. The latter is smaller than the former whenever capacity constraints bind before the merger and put upward pressure on prices.

The median price effects are 2.7, 2.1, 8.1, and 4.8 percent under logit, linear, log-linear, and AIDS demand respectively. The relative size of these simulated price effects are consistent with those found by Miller et al. (2016 and 2017) and Crooke et al. (1999). The greater curvature of the log-linear and AIDS systems generates larger price effects, all else equal.

Note that standard $GUPPI$ is strictly positive because margins and diversion are strictly positive. Firm 1's $ccGUPPI$, however, is zero more than 25 percent of the time. In these instances, per proposition 1, firm 1 is constrained before and after the merger, and sets prices where demand sells out capacity.

Table 2 shows upward price pressure and simulated price effects for firm 1 in each of the nine possible cases defined by the pre- and post-merger capacity constraints. In cases 9 and 5, where firm 1 is never constrained, $ccGUPPI$ and $GUPPI$ are identical. Consistent with the results of Miller et al. (2017), the median upward price pressure is close the median linear and logit price effects and smaller than the median AIDS and log-linear price effects.

In cases 1 and 3, where product 1 is constrained before and after the merger, $ccGUPPI$ is zero reflecting the fact that the merger does not change the price-setting equation of product 1. There are, however, simulated price increases. The merged firm has an incentive to raise the price of product 2 because of the upward price pressure represented by θ . A higher price on product 2 then shifts demand for product 1, resulting in a higher equilibrium price for product 1.

In case 6, both firms are constrained before and after the merger, so $ccGUPPI$ equals zero and all demand systems predict a zero price effect.¹⁴ Standard $GUPPI$, which ignores the capacity constraints, has a positive median under case 6.

In the other cases (2, 4, 7, and 8) the median $ccGUPPI$ is close to those of the logit and linear price effects, and noticeably less than those of the AIDS and log-linear price effects. In addition, the median $ccGUPPI$ is less than that of standard $GUPPI$ because from former accounts for capacity constraints.

Figure 3 illustrates the empirical distribution of $ccGUPPI$, $GUPPI$, and the simulated price effects. The histograms confirm that the distribution of $ccGUPPI$ is similar to those of the linear and logit simulated price effects. The distributions of the AIDS and log-linear price effects also have the same general shape but much longer and thicker right tails. The distribution of standard $GUPPI$ clearly differs from all the others.

¹⁴We are finding a small number of price increases in case 6 under log-linear demand. We are working to understand why this is happening.

Table 2: Median ccGUPPI, GUPPI, and Simulated Price Effects

Case	ccGUPPI	GUPPI	Simulated Price Increase			
			Linear	Logit	AIDS	Log-Lin
1 $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$	0.00	0.05	0.01	0.01	0.02	0.03
2 $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	0.05	0.11	0.06	0.05	0.20	0.26
3 $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$	0.00	0.07	0.01	0.01	0.01	0.02
4 $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$	0.06	0.15	0.06	0.05	0.34	0.50
5 $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$	0.09	0.09	0.08	0.06	0.24	0.33
6 $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	0.00	0.08	0.00	0.00	0.00	0.00
7 $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$	0.04	0.13	0.03	0.02	0.07	0.15
8 $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$	0.05	0.08	0.05	0.03	0.06	0.12
9 $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.06	0.06	0.06	0.05	0.11	0.17

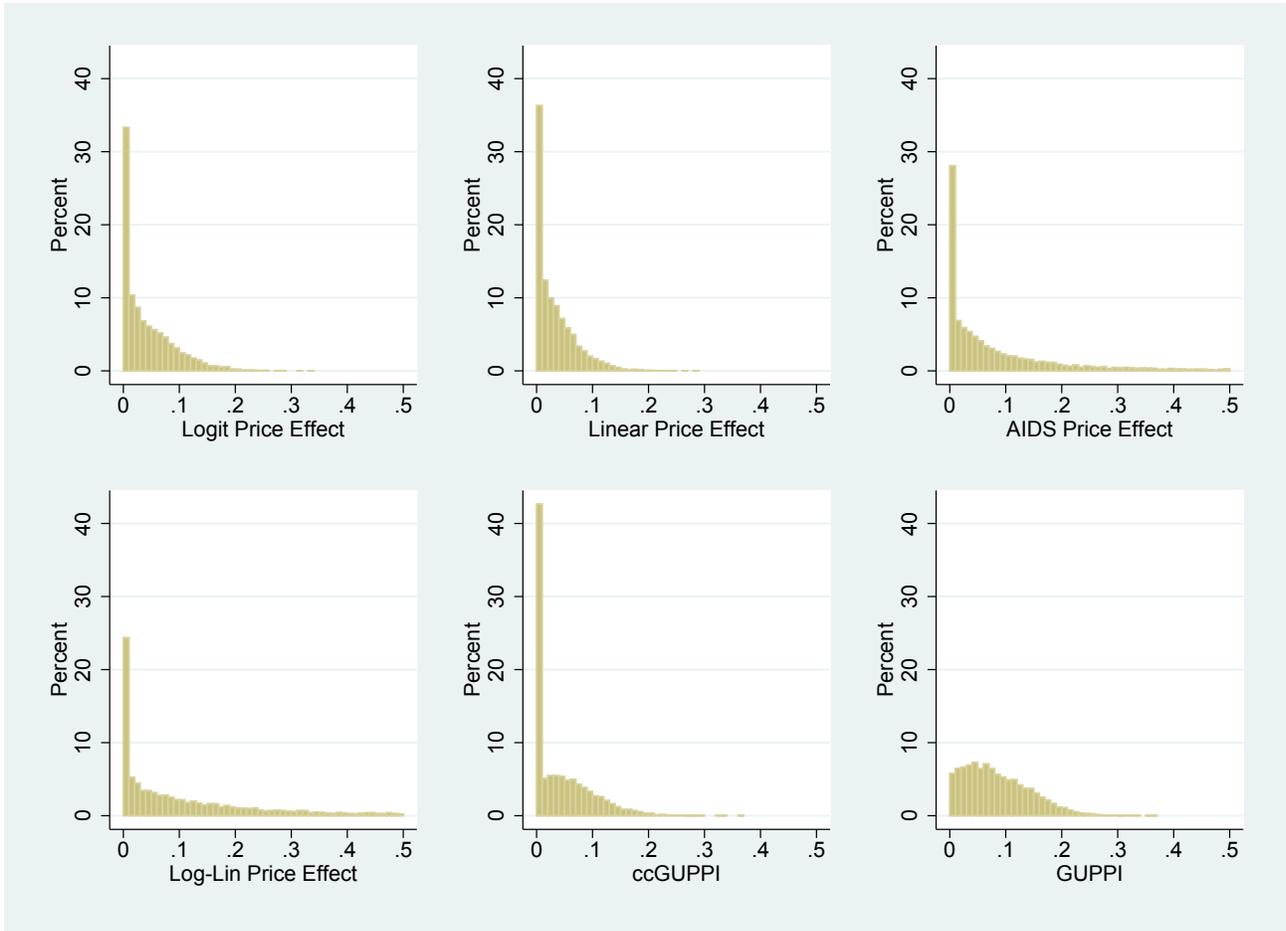


Figure 3: Distribution of ccGUPPI, GUPPI, and Actual Merger Price Effects

5.3 ccGUPPI as a predictor of price effects

We first evaluate *ccGUPPI*'s accuracy as a predictor of merger price effects graphically. The graphs in figure 4 each plot either *ccGUPPI* or *GUPPI* on the vertical axis and the simulated merger price effect under a specific demand systems on the horizontal axis. A 45 degree reference line indicates exact predictions, where *ccGUPPI* or *GUPPI* equals the simulated price increase.

Under logit and linear demand, *ccGUPPI* is quite accurate with the dots tightly dispersed around the 45 degree line. In contrast, standard *GUPPI* is systematically biased upward, with the dots dispersed above the reference line. The vertical line of dots along the vertical axis represents observations where the merging firms are constrained before and after the merger.

Under AIDS and log-linear demand *ccGUPPI* under-predicts simulated price effects. This is consistent with the results of Miller et al. (2016 and 2017) who find upward price pressure tends to under-predict price increases with AIDS and log-linear demand. In contrast, the *GUPPI* predictions are widely dispersed under AIDS and log-linear demand. This is because standard *GUPPI* tends to over-predict the actual price increase when capacity constraints bind tightly and under-predicts prices effects capacity constraints do not bind.

Table 3: Prediction Error

	Demand System			
	Linear	Logit	AIDS	Log-Lin
Median Prediction Error				
ccGUPPI	0.004	0.001	-0.016	-0.050
GUPPI	0.040	0.021	0.002	-0.024
Standard Deviation of Prediction Error				
ccGUPPI	0.023	0.013	0.242	0.359
GUPPI	0.050	0.052	0.254	0.367
Median Absolute Prediction Error				
ccGUPPI	0.011	0.003	0.016	0.050
GUPPI	0.038	0.022	0.047	0.068

We also evaluate *ccGUPPI* and *GUPPI*'s accuracy as a predictor of merger price effects numerically. Define the prediction error as *ccGUPPI* or *GUPPI* minus the analytically derived price increase under a specific demand system. Table 3 shows the median prediction error, the standard deviation of the prediction error, and the median absolute prediction error of *ccGUPPI* and *GUPPI* relative to each demand system.

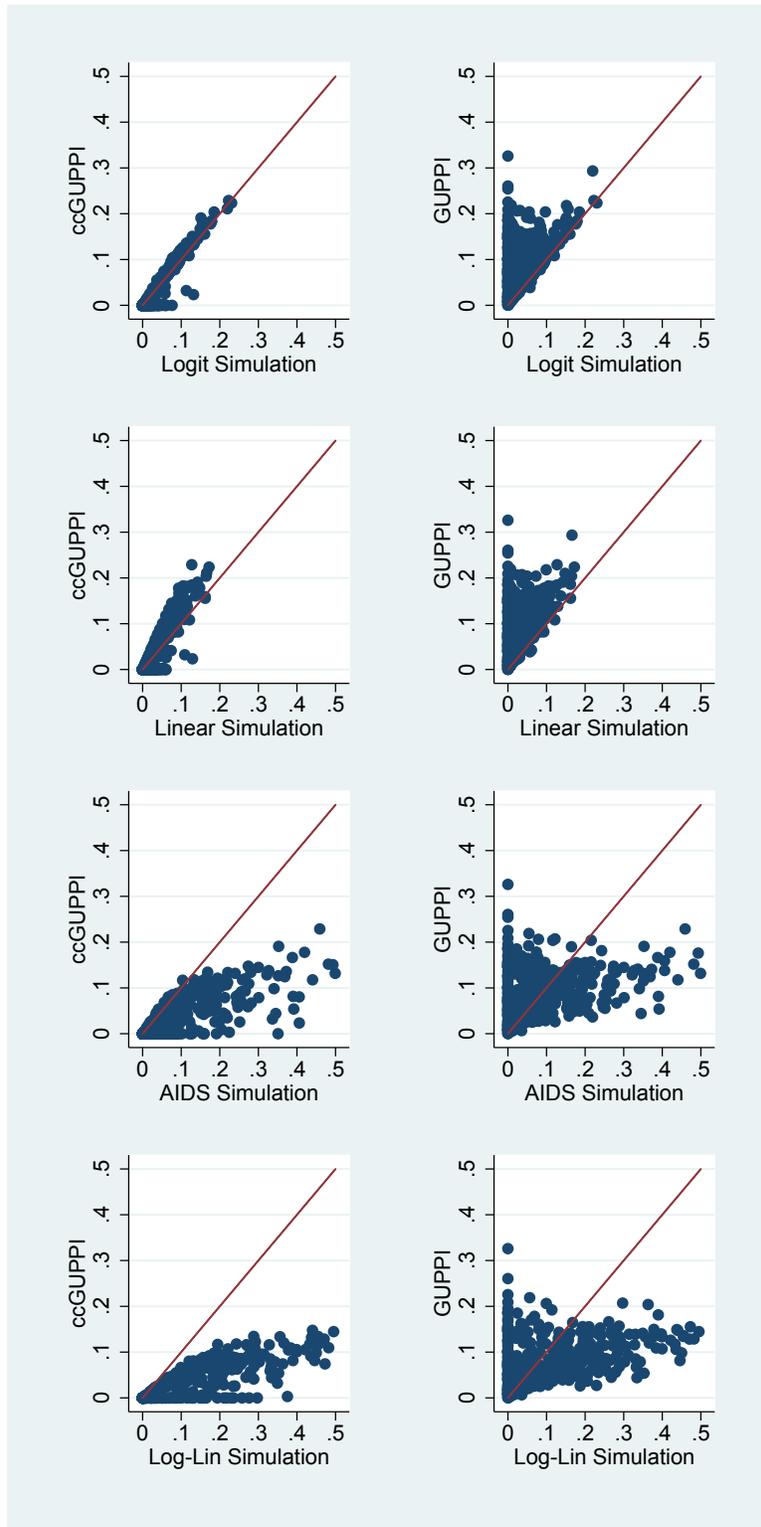


Figure 4: ccGUPPI and GUPPI price predictions (y-axis) versus simulated price effect (x-axis).

The median prediction error confirms that *ccGUPPI* is good predictor of price effects under linear and logit demand, but tends to under-predict price increases under AIDS and log-linear demand. In addition, the prediction error of *GUPPI* has a higher standard deviation than *ccGUPPI* under all demand systems. Perhaps more importantly, if the underlying demand is either linear, logit, AIDS, or log-linear, one can be confident that *ccGUPPI* is either relatively accurate (under linear or logit) or under-predicts price effects (under AIDS and log-linear). By comparison, there is no way to predict the likely sign of the prediction error with standard *GUPPI*.

Finally, the median absolute prediction error statistics again suggest that *ccGUPPI* out performs *GUPPI*. Under all demand systems, *ccGUPPI* has a lower median absolute error than standard *GUPPI*.

5.4 *ccGUPPI* as a merger screen

Proposition 1 shows that *ccGUPPI* can predict whether capacity constraints will eliminate merger price effects. If *ccGUPPI* equals zero for all of the merging firms products, one can conclude that price effects are unlikely, at least in the short run. Such a test is particularly powerful given it does not depend on the functional form of demand.

Antitrust agencies may also want to flag mergers whose price effects will likely be greater than a specified threshold. Follow Miller et al. we consider a test to screen out all mergers likely to generate a price increase greater than 10 percent, as predicted by *ccGUPPI* or standard *GUPPI*.

For each observation in the simulated data, we determine whether *ccGUPPI* and standard *GUPPI* exceed ten percent. A false positive, or Type II error, means that the *ccGUPPI* or *GUPPI* of at least one of the merging products is greater than ten percent while the actual price effect of both merging products is less than ten percent. A false negative, or Type I error, means that the *ccGUPPI* or *GUPPI* of both products is less than ten percent and the actual price effect of at least one product is greater than ten percent.

Table 4 summarizes the frequency of type I and type II errors. The prevalence of type I errors is clearly lower for *ccGUPPI* than standard *GUPPI*. This is obviously because standard *GUPPI* over-predicts price effects when firms are capacity constrained.

The prevalence of type II errors is lower for standard *GUPPI* than *ccGUPPI*, at least under AIDS and log-linear. This is in part explained by instances where standard *GUPPI* generates larger price effects because it does not account for capacity constraints. In essence, standard *GUPPI* generates fewer false positives by mistake, because it does not account for capacity constraints.

Overall, *ccGUPPI* generates substantially fewer total type I and II errors under logit, linear, and AIDS demand. Standard *GUPPI* generates slightly fewer total error under log-linear demand, 31

Table 4: Frequency of Merger Screen Errors

	Demand System			
	Linear	Logit	AIDS	Log-Lin
<i>ccGUPPI</i>				
Type 1 Error (False Positive)	17%	4%	0%	0%
Type 2 Error (False Negative)	0%	0%	17%	34%
Total Type 1 and 2 Errors	17%	4%	17%	34%
<i>GUPPI</i>				
Type 1 Error (False Positive)	49%	37%	24%	16%
Type 2 Error (False Negative)	0%	0%	8%	18%
Total Type 1 and 2 Errors	49%	37%	33%	34%

percent of the time versus 34 percent of the time for *ccGUPPI*.

6 Conclusion

This paper provides antitrust practitioners with a simple tool to evaluate mergers involving one or more capacity-constrained firms. Simulated data from our Monte Carlo experiments suggest that *ccGUPPI* performs better than standard *GUPPI*, and is a quite accurate predictor of merger price effects when demand is linear or logit, and a lower bound on price effects under AIDS or log-linear demand. We now briefly discuss two caveats.

First, capacity constraints are necessarily transitory. It is entirely appropriate for antitrust agencies to consider additional capacity that is about to become available. If the merging firms are deemed likely to build additional capacity in the near future, *ccGUPPI* can be used as part of a broader analysis that considers both short- and long-run effects. Notably, so long as the putative capacity expansion is not merger-specific, an increase in capacity exacerbates merger price effects for capacity-constrained firms. This is because such an increase would lower the constrained firm’s (long run) pre-merger price. In the language of the paper, this would decrease λ without affecting *GUPPI* or θ .

Second, implementing *ccGUPPI* requires one additional piece of information not required for a traditional *GUPPI* calculation. Specifically, one needs to know either the price elasticity of demand, or, equivalently, the difference between marginal revenue or marginal costs (λ). Identifying the price elasticity of demand econometrically is difficult given it requires exogenous variation in price. Doing so when firms are capacity constrained can be even more challenging given some consumers

may face a truncated choice set (see for example Conlon and Mortimer (2013)). Nevertheless, antitrust agencies can supplement econometric evidence with deposition testimony, data, or documents from industry participants regarding the price sensitivity of demand. Consumers' stated preferences and profit-maximizing firms' understanding of consumer preferences may provide valuable supplementary evidence. Finally, natural experiments such as unexpected supply disruptions that generate exogenous variation in the merging firms' prices might allow the agencies to identify the price elasticity of demand short of full demand system estimation.

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