INTRODUCTION

• Goal: Develop a rigorous method to integrate data from different sources—full system and subsystems—for optimal parameter estimation and computable uncertainty bounds

• Assumption: Study the identification of parameters for the case where the subsystems are lognormally distributed and the full system is normally distributed

• Methodology: Use the maximum likelihood estimation (MLE) technique to derive the formal convergence results on the MLEs to the true subsystem and full system values as well as the asymptotic normal distributions for the MLEs

• Application: Apply the method on the cold-formed steel (CFS) structural system, where the fasteners are viewed as the subsystems and the shear wall is viewed as a full system.

• Result: Use the data independently collected from multiple sources to provide a much better estimation of the shear wall strength when compared with the previous simple one-sample estimation method [1]

BACKGROUND

• The cold-formed steel (CFS) structural systems are commonly used for low mid-rise construction.

• Each fastener system (binary) matches the physical intuition.

• Only based on shear wall experimental test data.

• Subsystems (fastener) distribution: Lognormal

• Full system (shear wall) distribution: Normal

MODEL

Model Assumptions

• Full system (shear wall) distribution: Normal ($\mu_s, \sigma^2_s$)

• Subsystems (fastener) distribution: Lognormal ($\mu, \sigma^2)$, where $\sigma^2 = 0.13$ per American Iron and Steel Institute S-100 [4]

• Connection between full system and subsystems: $\mu_s = h_1(\mu)$ and $\sigma^2_s = h_2(\sigma^2)$

MLE Formulation

• Full system data: $Y_1, \ldots, Y_n$ parameters $\eta = [\mu, \sigma^2]^T$

• Subsystems data: $X_{ij}, i = 1, \ldots, n$ parameter $\theta = \theta_s$

Log-Likelihood Function

Assume data are independent and use the canonical exponential family representation:

$$L(\theta) = \prod_{i=1}^{n} \left( \frac{1}{2\sigma^2} \right)^{\frac{n_i}{2}} e^{-\frac{1}{2\sigma^2} \sum_{i=1}^{n} \frac{Y_i - \mu}{\sigma^2}} \text{constant}$$

Connection: $\mu_s = h_1(\mu)$ and $\sigma^2_s = h_2(\sigma^2)$

MLE definition

$$\hat{\theta} = \arg \max L(\theta) \text{ and } \hat{\eta} = [\hat{\mu}, \hat{\sigma}^2]$$

Consistency

Under some regularity conditions

- $B = \eta^0$ in probability as $n + m \rightarrow \infty$

- $B = \theta^0$ in probability as $n + m \rightarrow \infty$

Asymptotic Normality

Under some regularity conditions

- $\sqrt{n}(\hat{\eta} - \eta^0)$ has a normal distribution as $n + m \rightarrow \infty$

- $\sqrt{n}(\hat{\theta} - \theta^0)$ has a normal distribution as $n + m \rightarrow \infty$

Confidence intervals are readily available and the MLE results can be generalized into other distributions.

SUBSYSTEM TEST DATA

(a) subfastened shear wall

(b) shear wall with deflected shape

Figure 4: Shear wall simulation results generated by the software OpenSees, where the microstructurality ranges the physical structure.

NUMERICAL RESULTS

• More information leads to more accurate estimate

• New method improves the accuracy significantly

Shear Wall Strength

\begin{tabular}{|c|c|c|}
\hline
Mean Estimate & 3.19 & 2.99 \\
95% C.I. & (3.17, 3.20) & (2.91, 3.07) \\
\hline
\end{tabular}

Table 1: Shear wall strength estimation and 95% confidence interval calculated by the proposed method using the shear wall data and the fastener data, and by the single sample mean method using only the shear wall data.

CONCLUSIONS

• This work studies the convergence and asymptotic results on system identification with log-normal distributed subsystems and apply the results to cold-formed steel shear wall system.

• By integrating the information from different sources, the accuracy of the estimates can be significantly improved.

FUTURE WORK

• More experimental data for better estimation

• What data is the most valuable?

• Extend method to more general settings (ongoing work)

• More general distributions?

• Time-varying input variables?

• Optimal system design

• How to arrange subsystems to maximize overall reliability?

• Adopt Bayesian framework

• How to incorporate user-specified prior distributions?

REFERENCES


CONTACT

Long Wang, Applied Mathematics and Statistics, Johns Hopkins University
Office: 203, Whitake Hall, Johns Hopkins University
Email: long.wang@jhu.edu
Website: http://pages.jhu.edu/~lwang155/